

**AP Review Topics**  
(topic with \* are BC only)

Section 2.2

One-sided limits versus two-sided limits:  $\lim_{x \rightarrow a} f(x) = L$  iff  $\lim_{x \rightarrow a^+} f(x) = L = \lim_{x \rightarrow a^-} f(x)$

Section 2.3

Limit properties (table on page 110)

Section 2.4

Continuity Test:  $f(x)$  is continuous at  $x = a$  iff  $\lim_{x \rightarrow a} f(x) = f(a)$ .

Types of discontinuities:

- i) removable – limit exists, but not equal to function value (hole in graph)
- ii) infinite – one-sided limit approaches infinity (vertical asymptote in graph)
- iii) jump – one-sided limits approach different values (break in graph)

Removing removable discontinuities

- i) Find limit at  $x = a$ .
- ii) Define or redefine function value to be that limit at  $x = a$ .

Limit composition property:  $\lim_{x \rightarrow a} f(g(x)) = f\left(\lim_{x \rightarrow a} g(x)\right)$

Section 2.5

Definitions of vertical and horizontal asymptotes

HA: If  $\lim_{x \rightarrow \infty} f(x) = L$  or  $\lim_{x \rightarrow -\infty} f(x) = L$ , then  $y = L$ .

VA: If  $\lim_{x \rightarrow a^+} f(x) = \pm\infty$  or  $\lim_{x \rightarrow a^-} f(x) = \pm\infty$ , then  $x = a$ .

End-behavior of rational functions

- i) If numerator's degree is greater than denominator's, then infinite end-behavior (either  $-\infty$  or  $+\infty$ , so check signs).
- ii) If denominator's degree is greater than the numerator's, then behavior approaches  $y = 0$ .
- iii) If numerator's and denominator's degrees are equal, then behavior is  $y = p/q$  where  $p/q$  is the ratio of lead coefficients.

## Section 2.6

Concepts of average and instantaneous rates of change

$$\text{Average: } \frac{f(b) - f(a)}{b - a} \quad \text{or} \quad \frac{f(a+h) - f(a)}{h}$$

$$\text{Instantaneous: } \lim_{\Delta x \rightarrow 0} \frac{f(b) - f(a)}{b - a} \quad \text{or} \quad \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

## Section 2.7

Definition of the numerical derivative as an instantaneous rate of change

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

## Section 2.8

$$\text{Definition of the function derivative: } f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

A function is differentiable at a point if:

- i) it is continuous (see Section 2.4),
- ii) its graph has no corner (the one-sided derivatives are different),
- iii) its graph has no cusp (the one-sided derivatives change from  $-\infty$  to  $+\infty$ ), or
- iv) its graph has no vertical tangents (the derivative is  $\pm\infty$ ).

## Section 2.9

Tangent line approximation:  $f(x) \approx T(x) = f(a) + f'(a)(x - a)$  near  $a$ .

## Section 2.10

Basic graph behavior on an interval:

- i) If  $f$  is increasing,  $f'$  is positive.
- ii) If  $f$  is decreasing,  $f'$  is negative.
- iii) If  $f$  has upward concavity,  $f'$  is increasing and  $f''$  is positive.
- iv) If  $f$  has downward concavity,  $f'$  is decreasing and  $f''$  is negative.

Definition of inflection point: point on the graph where concavity changes

Antiderivatives:  $F'(x) = f(x)$

## Section 3.2

### Product and Quotient Rules

$$\text{i) } \frac{d}{dx} [f(x) \cdot g(x)] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$\text{ii) } \frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$$

## Section 3.3

Position & velocity & acceleration:  $s''(t) = v'(t) = a(t)$

### Speed

i) Magnitude of velocity:  $speed = |v(t)|$

ii) Speed is increasing when velocity and acceleration have the same sign.

iii) Speed is decreasing when velocity and acceleration have different signs.

Distance traveled and displacement (see Section 5.3)

## Section 3.4

Derivatives of common trigonometric functions (table of page 222)

## Section 3.5

### Chain Rule

$$\text{i) } (f \circ g)'(x) = f'(g(x)) \cdot g'(x)$$

$$\text{ii) } \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Derivative of exponential functions:  $\frac{d}{dx} a^x = a^x \cdot \ln a$

\*Slope of parametric curves:  $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

## Section 3.6

Implicit differentiation – first and second derivatives

Inverse trigonometric derivatives:

$$\text{i) } \frac{d}{dx} \arcsin(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\text{ii) } \frac{d}{dx} \arctan(x) = \frac{1}{1+x^2}$$

## Section 3.7

Derivatives of logarithmic functions:

$$\text{i) } \frac{d}{dx} \ln x = \frac{1}{x}$$

$$\text{ii) } \frac{d}{dx} \log_a x = \frac{1}{x \cdot \ln a}$$

Logarithmic differentiation – use when base and exponent have variables

- i) “log” both sides and use logarithmic power rule.
- ii) Differentiate implicitly.
- iii) Solve for  $dy/dx$  and substitute back in for  $y$ .

$$\text{Definition of } e: \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = \lim_{x \rightarrow 0} (1+x)^{1/x} = e$$

## Section 3.8

Using tangent lines to predict other values:  $f(a) \approx T(a)$  (see Section 2.9)

Differentials:  $dy \approx \Delta y$

Definitions of error:

$$\text{i) } \text{Actual error: } |f(x) - T(x)| = \Delta y \approx dy$$

$$\text{ii) } \text{Relative error: } \frac{|f(x) - T(x)|}{f(x)} = \frac{\Delta y}{y} \approx \frac{dy}{y}$$

## Section 4.1

Related rates – derivatives with respect to time

## Section 4.2

Definitions of absolute and relative maximum and minimums and critical points

How to find absolute extreme values:

- i) Test all critical point values.
- ii) Test endpoints.

## Section 4.3

Mean Value Theorem:  $f'(c) = \frac{f(b) - f(a)}{b - a}$ , where  $a < c < b$

First Derivative Test

- i) If  $f$  changes from increasing to decreasing, there is a relative maximum.
- ii) If  $f$  changes from decreasing to increasing, there is a relative minimum.

Second Derivative Test

- i) If  $f'(a) = 0$  and  $f''(a) > 0$ , then there is a relative minimum at  $x = a$ .
- ii) If  $f'(a) = 0$  and  $f''(a) < 0$ , then there is a relative maximum at  $x = a$ .

## Section 4.5

l'Hôpital's Rule:  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$  when  $f(x) = g(x) = 0$  or they are both  $\pm\infty$

(the original limit yields  $0/0$  or  $\infty/\infty$ ).

Other indeterminate forms:  $0 \cdot \infty$ ,  $\infty - \infty$ ,  $1^\infty$ ,  $0^0$ ,  $\infty^0$ .

- i)  $0 \cdot \infty$ : reciprocate one factor to get  $0/0$  or  $\infty/\infty$ .
- ii)  $\infty - \infty$ : find a common denominator to force  $0/0$  or  $\infty/\infty$ .
- iii) \*for all other forms, use logarithms:
  - a) Set limit equal to  $\lim_{x \rightarrow a} y$ .
  - b) "log" both sides and use logarithmic power rule.
  - c) Evaluate limit.
  - d) "e" both sides.

## Section 4.6

### Optimization

- i) Find function to be maximized or minimized.
- ii) Use secondary equation to get in terms of one variable.
- iii) Differentiate and find critical points (check endpoints)

## Section 4.9

General antiderivatives (+C) & particular antiderivatives (initial condition for known C)

Definition and solving basic differential equations (initial-value problems).

Antiderivative patterns (table of page 330)

### Slope Fields

- i) Creating fields from a given differential equation
- ii) Using fields to approximate particular antiderivatives

## Section 5.1

Concept and approximations of Riemann sum

## Section 5.2

Left, middle, right, upper, and lower bound approximations.

## Section 5.3

Definite versus indefinite integrals

Indefinite integral patterns (table on page 372)

Total Change Theorem (FTC)

- i) Displacement:  $\int_{t_1}^{t_2} v(t) dt$
- ii) Distance traveled:  $\int_{t_1}^{t_2} |v(t)| dt$
- iii) New Position:  $s(t_2) = s(t_1) + \int_{t_1}^{t_2} v(t) dt$

## Section 5.4

Definition of integral functions:  $g(x) = \int_a^x f(t) dt$

Fundamental Theorem of Calculus

i) If  $g(x) = \int_a^x f(t) dt$ , then  $g'(x) = f(x)$ .

ii)  $\int_a^b f(x) dx = F(b) - F(a)$

## Section 5.5

$u$ -substitution technique

i) Rewrite bounds in terms of  $u$  in definite integrals.

ii) Watch for extra  $x$  factors that can be rewritten using the original substitution.

iii) Dead giveaway: you see a function and its derivative.

Antiderivative of tangent function:  $\int \tan x dx = \ln |\sec x| + C$

## Section 5.6

\*Integration by parts:  $\int u dv = uv - \int v du$

\*Table method for integration by parts (not in book, see notes or website)

\*Antiderivative of natural logarithmic function:  $\int \ln x dx = x \ln x - x + C$

## Section 5.7

$\int \sin^m x \cdot \cos^n x dx$ , when  $m$  and  $n$  are even or odd.

i) If  $m$  and  $n$  are both even, use half-angle formula.

ii) Otherwise, use Pythagorean Identity and  $u$ -substitution.

Integrating rational functions

i) Use long division when numerator's degree on top is same as or greater than the denominator's

ii) The "+1-1" magic trick can sometimes be used instead of long division.

iii) \*Partial fraction decomposition – when denominator has linear factors and degree on top is smaller

iv)  $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a} + C$  and  $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$

## Section 5.9

$$\text{Trapezoidal Rule: } \int_a^b f(x) dx \approx \frac{\Delta x}{2} [f(x_0) + 2 \cdot f(x_1) + \cdots + 2 \cdot f(x_{n-1}) + f(x_n)]$$

## Section 5.10

\*Improper integrals

$$\text{i) } \int_a^\infty f(x) dx = \lim_{b \rightarrow \infty} \int_a^b f(x) dx$$

$$\text{ii) } \text{Unbounded at } x = b: \int_a^b f(x) dx = \lim_{c \rightarrow b^-} \int_a^c f(x) dx$$

## Section 6.1

Area between two curves:

$$\text{i) } \int_a^b [f(x) - g(x)] dx, \text{ where } f \text{ is the top curve and } g \text{ is the bottom curve.}$$

$$\text{ii) } \int_a^b [f(y) - g(y)] dy, \text{ where } f \text{ is the right curve and } g \text{ is the left curve.}$$

$$\text{*Area of parametrically-defined regions: } \int_a^b y(t) \cdot x'(t) dt \text{ or } \int_a^b x(t) \cdot y'(t) dt$$

## Section 6.2

Volume of solids with known cross-sections.

$$\text{i) } \text{Washer method: } \pi \int_a^b [R(x)]^2 - [r(x)]^2 dx, \text{ where } R \text{ is the outer radius from the rotation axis and } r \text{ is the inner radius.}$$

$$\text{ii) } \text{Cross-section method: } \int_a^b A(x) dx, \text{ where } A \text{ is the area of any cross-section.}$$

iii) Watch for rotation axes other than  $y = 0$  and  $x = 0$ .

## Section 6.3

$$\text{*Arc length: } L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

You might need to write functions in parametric form:  $x = t$  and  $y = f(t)$ .

## Section 6.4

$$\text{Average value of a function: } \text{Avg} = \frac{1}{b-a} \int_a^b f(x) dx$$

## Section 7.1

Modeling with differential equations (“rate is proportional to...”)

## Section 7.2

Slope fields (see Section 4.9)

\*Euler’s Method – make table with  $x$ ,  $y$ ,  $y'$ ,  $hy'$ , and  $y + hy'$

## Section 7.3

Separable differential equations:  $\frac{dy}{dx} = f(x) \cdot g(y)$

- i) Separate the variables.
- ii) Integrate both sides.
- iii) Find the constant of integration.
- iv) Solve for  $y$ .

## Section 7.4

Exponential growth/decay

- i) DE:  $\frac{dy}{dt} = ky$  (growth when  $k > 0$ , decay when  $k < 0$ )
- ii) Solution:  $y(t) = Ce^{kt}$ , where  $C$  is the initial value  $y_0$ .

Half-life:  $HL = \frac{-\ln 2}{k}$

## Section 7.5

\*Logistic growth

- i) DE:  $\frac{dy}{dt} = ky \left(1 - \frac{y}{M}\right)$ , where  $k > 0$  and  $M$  is the carrying capacity
- ii) Solution:  $y(t) = \frac{M}{1 + Ce^{-kt}}$ , where  $C = \frac{M - y_0}{y_0}$
- iii) Don’t forget to rewrite if in the form  $\frac{dy}{dt} = k_1 y(M - y)$ .

Logistic curve and behavior

- i) POI between  $y = 0$  and  $y = M$  (greatest rate of increase)
- ii)  $\lim_{t \rightarrow \infty} y(t) = M$

## Section 8.1

\*Definition of a sequence,  $a_n = f(n)$ .

$$*\lim_{n \rightarrow \infty} a_n = \lim_{x \rightarrow \infty} f(x)$$

## Section 8.2

\*Definitions of infinite series (differences between sequences and series), partial sums, convergent, divergent

\*Geometric series:  $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$ , when  $|r| < 1$ , diverges otherwise

\*Harmonic series:  $\sum_{n=1}^{\infty} \frac{1}{n}$ , diverges

\*Divergence Test

## Section 8.3

\*Integral Test

\* $p$ -series Test

\*Direct Comparison Test

\*Limit Comparison Test

## Section 8.4

\*Alternative Series Test

\*Alternating Series Estimation Theorem:  $|R_n| = |s - s_n| \leq b_{n+1}$

\*Absolute Convergence Test

\*Ratio Test

## Section 8.5

\*Definitions of power series, interval of convergence, radius of convergence

\*Power series converge at a point, for all real numbers, or on some interval that is symmetrical about the center (except possibly at the endpoints).

\*Testing for intervals of convergence:

i) If the series is geometric, solve  $|r| < 1$  (no need to check endpoints).

ii) Use the Ratio Test for all others: solve  $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$  (check endpoints separately).

## Section 8.6

\*Operations involving power series (addition, subtraction, multiplication, division)

\*Composition with power series: If  $f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n$  where  $|x-a| < R$ , then

$$f(g(x)) = \sum_{n=0}^{\infty} c_n (g(x)-a)^n \text{ where } |g(x)-a| < R.$$

\*Differentiation and integration of power series

i) The radius of convergence is the same (but the endpoints can change).

ii) If the first term in the power series is constant, don't forget to adjust the initial index value.

iii) Don't forget the  $+C$  term when integrating.

## Section 8.7

\*Taylor and Maclaurin series – coefficients are  $c_n = \frac{f^{(n)}(a)}{n!}$

\*Taylor polynomials – partial sums of Taylor series (see Section 8.9)

\*Taylor Inequality to find the Lagrange error bound:

$$|R_n(x)| \leq \frac{M}{(n+1)!} |x-a|^{n+1}, \text{ where } |f^{(n+1)}(x)| \leq M$$

\*Common power series (see table on page 618)

## Section 8.9

\*Using Taylor polynomials to approximate function values:  $f(a) \approx T_n(a)$

\*Error bound analysis using Taylor polynomials

- i) Graphical -  $|f(x) - T_n(x)|$
- ii) Alternating Series Estimation Theorem (see Section 8.4)
- iii) Taylor's Inequality (see Section 8.7)

## Section 10.4

\*Planar motion with vectors (only 2D, see notes)

\*Position, velocity, and acceleration:  $\vec{r}''(t) = \vec{v}'(t) = \vec{a}(t)$

\*Speed

- i)  $|\vec{v}(t)| = \sqrt{[x'(t)]^2 + [y'(t)]^2}$
- ii) Use the derivative of speed to test when it is increasing or decreasing.

\*Distance traveled and displacement

- i) Distance traveled (arc length):  $\int_a^b |\vec{v}(t)| dt$  (result is a scalar)
- ii) Displacement:  $\int_a^b \vec{v}(t) dt$  (result is a vector)
- iii) Position:  $\vec{r}(b) = \vec{r}(a) + \int_a^b \vec{v}(t) dt$  (result is a vector)

## Section H.1

\*Definition of and conversions involving the polar coordinate system

\*Common graphs – circles, roses, and limaçon (cardioid)

\*Slope of a polar curve:  $\frac{dy}{dx} = \frac{\frac{d}{d\theta}(r \cdot \sin \theta)}{\frac{d}{d\theta}(r \cdot \cos \theta)}$ , where  $r(\theta)$  is the curve

## Section H.2

\*Area of a polar region:  $A = \frac{1}{2} \int_a^b [r(\theta)]^2 d\theta$