Writing Conclusion Statements for Convergence Tests

Some have asked for clarification on how to properly answer problems concerning the convergence or divergence of an infinite series based on the test used. Below are examples to illustrate just how to do so based on instructor and AP Exam preferences.

Please note that these examples given show only the conclusion statement… in many cases there will be other work that justifies your conclusion.

Geometric Series

$\sum_{n=1}^{\infty} 2^{2n} 3^{1-n}$ diverges since it is a geometric series with $|r| = \frac{4}{3} \geq 1$.

Note: Your work should show the geometric series in the form $\sum_{n=1}^{\infty} ar^{n-1}$.

p-series

$\sum_{n=1}^{\infty} \frac{5}{n^3}$ converges since it is a p-series with $p > 1$.

Note: It is okay to treat $c\sum_{n=1}^{\infty} \frac{1}{n^p}$ as a p-series.

Divergence Test

$\sum_{n=1}^{\infty} \frac{n^2}{5n^2+4}$ diverges by the Divergence Test since $\lim_{n\to\infty} \frac{n^2}{5n^2+4} = \frac{1}{5} \neq 0$.

Integral Test

$\sum_{n=1}^{\infty} \frac{\ln n}{n}$ diverges by the Integral Test since $\frac{\ln n}{n}$ is eventually positive and decreasing and $\int_{1}^{\infty} \frac{\ln x}{x}\,dx$ diverges.

Note: The Integral Test requires that $f(x)$ be continuous, but that is understood given the improper integral's evaluation (which needs to be shown in your work).

Comparison Test

$\sum_{n=1}^{\infty} \frac{1}{2^n+1}$ converges by the Comparison Test since $\frac{1}{2^n+1} \le \frac{1}{2^n}$ and $\sum_{n=1}^{\infty} \frac{1}{2^n}$ converges.

Note: While it never hurts to do so, you do not have to include why the series being compared to is convergent/divergent.

Limit Comparison Test

$\sum_{n=1}^{\infty} \frac{1}{2n+1}$ diverges by the Limit Comparison Test since $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges and

$$\lim_{n\to\infty} \frac{\left(\frac{1}{2n+1}\right)}{\left(\frac{1}{n}\right)} = \frac{1}{2} > 0 .$$

Note: While it never hurts to do so, you do not have to include why the series being compared to is convergent/divergent.

Alternating Series Test

$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ converges by the Alternating Series Test since $\frac{1}{n}$ is eventually decreasing and $\lim_{n\to\infty} \frac{1}{n} = 0$.

Note: Your work should show the alternating series in the form $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$.

Absolute Convergence Test

$\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$ is absolutely convergent (and therefore convergent) since $\sum_{n=1}^{\infty} \left| \frac{\cos n}{n^2} \right|$ converges.

Note: Your work should justify that $\Sigma|a_n|$ converges.

Ratio Test

$\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{3^n}$ is absolutely convergent (and therefore convergent) since

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+1} \frac{(n+1)^3}{3^{n+1}}}{(-1)^n \frac{n^3}{3^n}} \right| = \frac{1}{3} < 1$$