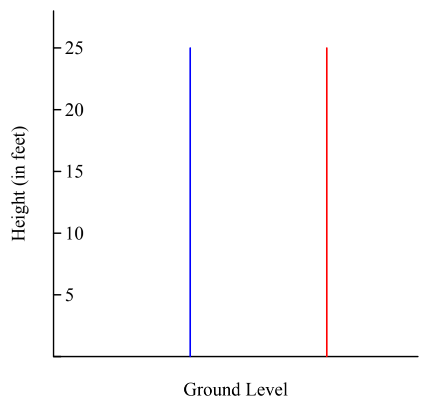
Analyzing a Falling Object

An Investigation of Rectilinear Motion

Introduction

In this lab, you will explore the rates at which an object falls. For example, consider a ball that is dropped with no initial velocity from a height of 25 feet. How long will it take the falling ball to reach ground level and what will its velocity be when it gets there? At constant time intervals, will the ball fall at a constant rate as in the blue graph in Fig 1, or will it fall at varying rates as in the red graph (click the graph below to animate)? To answer these questions, you need to understand how an object falls.

Fig 1



A function *y* that gives the position of an object as a function of time *t* is a **position function**. The position of a free-falling object (neglecting air resistance) under the influence of gravity can be modeled by the quadratic equation

$$y(t) = \frac{1}{2} \cdot g \cdot t^2 + v_0 \cdot t + y_0$$

where y_0 is the initial height, v_0 is the initial velocity of the object, and *g* is the acceleration due to gravity. On Earth, the acceleration due to gravity is approximately -32 feet per second per second.

To find the rates of the object at different times, you can find its **velocity function**. The velocity function is the **derivative** of the position function, that is $v(t) = y'(t)$. As you study derivatives and their uses, you will learn a technique called the power rule. Using that technique, you can detemermine a function in terms of *g*, *t*, and v_0. Use the position function above to derive the velocity function.

(4pts)

Depending on the initial velocity of a falling object, the subsequent velocities of the object while it is falling can be negative, positive, or zero. Describe the motion of the object when $v < 0$, $v = 0$, and $v > 0$.

(6pts)

In this investigation, you will model the position function of a falling object, then use it to *verbally*, *numerically*, and *analytically* find the velocities of the falling object.

The Experiment

The equipment for this experiment will be set up prior to beginning. When prompted, follow these procedures to perform the experiment:

A. Click the Quick Data Tool button above and select the Motion Sensor probe. Set the experiment for 100 samples with each one collected every 0.05 seconds. The range values should fall within the range [0, 8] when the units are selected to be in feet (ft). Call the lists Time1 and Dist1. Choose the Get After Collection option in the Select Data Collection Style menu, and leave all other fields at the default setting.

B. With one person holding the ball approximately 6 feet above the ranger, click Run to begin the collecting data process. Once the clicking from the motion detector starts, drop the ball. A second person should catch the ball as close to the ranger as possible, but avoid hitting the ranger!

C. After all the data has been collected, you should save the data and graph to this document under step 1.

Recording the Data

1. Perform the experiment described above. Save the data lists and graph from the experiment here.

(Raw data goes here.)

2. The data collected contains extraneous points from the ball's fall. In the Data Editor window, click the Select Data button to choose the data in the Time1 and Dist1 lists for the free fall only. Resave the data as Time2 and Dist2. You can make these adjustments in the data list in step 1.

3. The contents of Time2 needs to be adjusted so that the first entry is zero. To do this, create a new list called Time and enter the formula Time1-Time1[1]. Copy the data in Dist1 to create a list called Height. Save the adjustments, and display the Time and Height lists here (you do not need to show all of the data since the list is so long).

4. <u>Create a scatter plot that represents the height data with respect to adjusted time data.</u>

(8pts)

5. Using the scatter plot in step 4, <u>*verbally* describe how the object fell.</u>

(6pts)

6. Based on your response in step 5, <u>which of the two animations in Fig 1 is most representative of the data obtained in this experiment? Explain.</u>

(4pts)

Analyzing the Data

7. Using the statistical features of TI-InterActive!, <u>perform a quadratic regression analysis of the height and time data.</u> Use a Math Box, and name the regression model "h_reg". Show the calculations, and <u>rewrite the resulting model equation using appropriate variables.</u>

(6pts)

8. Create a graph of the model equation along with the scatter plot from step 4.

(4pts)

9. How does the graph of the model equation compare to the scatter plot?

(4pts)

10. The model found in step 7 is the position function of the falling object. Discuss how the values of *a*, *b*, and *c* correspond to the acceleration to due gravity, the initial height of the object, and the initial velocity of the object. Are the values of *a*, *b*, and *c* reasonable?

(12pts)

11. What are some of the physical factors that may have affected the experiment and caused the values in the model equation to vary from the theoretical equation?

(4pts)

12. Use the CAS features of TI-InterActive! to find the function that models the velocity of the object in the experiment. Define the velocity function as "v_reg" and display it using appropriate variables.

(6pts)

13. Use the data from the experiment to find the average velocities at five different points in time, then calculate the instantaneous velocities for the same points. Describe your calculations, then enter your results into a spreadsheet.

(10pts)

14. Compare the values of the average velocities and the instantaneous velocities. What observations can you make?

(4pts)

Extensions

15. Repeat the experiment, but this time start by tossing the ball up and catching it on the way down. Adjust the data as before, then create a scatter plot of the data.

(6pts)

16. Perform a quadratic regression analysis of the data found in step 15. Name the function "h_reg2".

(6pts)

17. How does the position function in step 16 compare to the one found in the original experiment?

(4pts)

18. Find the velocity function of the second experiment. Describe when the velocity is positive, negative, and zero.

(6pts)