

Advanced Algebra & Trigonometry
2nd Semester Exam Review

The final exam will consist of 50 multiple-choice questions taken from the topics presented during the spring semester only. The exam will be broken up into five sections, listed below with corresponding textbook sections:

- Sequences and Series (14%): Sections 11-1, 11-2, 11-3
- Right Triangle Trigonometry and Equations (30%): Sections 5-1, 5-2, 5-3, 6-5
- Trigonometric Functions (24%): Sections 5-5, 5-6
- Trigonometric Applications (18%): Sections 5-8, 7-1, 7-2
- Vectors (14%): Section 7-6

The types of problems that follow are representative of those on the actual exam (these are free-response instead of multiple-choice, however). It is suggested that you also study previous unit tests and quizzes, including the 3rd Quarter Exam (available in class only).

Answers to these problems are at the end of the document.

Sequences and Series

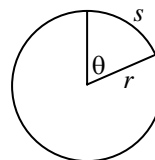
1. Write a function to represent the sequence $a_n = 14, 8, 2, -4, \dots$
2. Write a function to represent the sequence $a_n = 12, -18, 27, -40.5, \dots$
3. An arithmetic sequence has terms $a_1 = -1$ and $a_3 = 4$. Find the 10th term, a_{10} .
4. A geometric sequence has terms $a_1 = 64$ and $a_2 = -40$. Find the 7th term, a_7 .
5. Given the sequence $a_n = 4n + 3$, find the 15th partial sum, S_{15} .

6. Evaluate $\sum_{n=1}^6 3(2)^{n-1}$.

7. Evaluate $\sum_{k=1}^{\infty} 6\left(\frac{2}{5}\right)^k$.

Right Triangle Trigonometry and Equations

8. Evaluate $\csc(\pi/6)$.
9. Evaluate $\cos(-120^\circ)$.
10. Evaluate $\cot(\pi/2)$.
11. Convert 165° to radians.
12. Convert $\pi/12$ to degrees.
13. Use your calculator to evaluate $\sec(0.731)$.
14. Use your calculator to solve $\sin \theta = 0.488$, where θ is in degrees.
15. If $\sin \theta = -4/5$ and $\cos \theta = -3/5$, find $\tan \theta$.
16. If $\theta = 35^\circ$ and $r = 9$, approximate s (see illustration at the right).

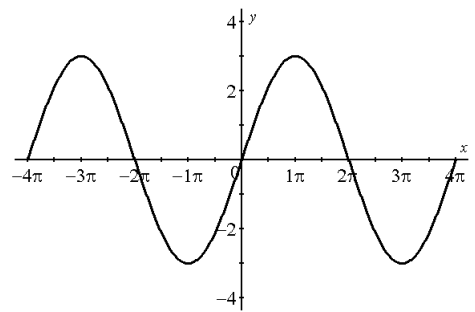


Right Triangle Trigonometry and Equations (continued)

17. In a right triangle, the hypotenuse is 23 centimeters and the shorter of the two legs is 6 centimeters. Find the measure of the smallest angle in the triangle.
18. A woman standing 200 feet from the base of a cliff measures the angle of elevation from the ground to the top of the cliff to be 63° . To the nearest foot, how tall is the cliff?
19. Solve $2 \cos x + \sqrt{3} = 0$ on the interval $[0, 2\pi)$.
20. Solve $\tan x = 0$ on the interval $[0, 2\pi)$.
21. Solve $\sin(2x) = -\frac{\sqrt{2}}{2}$ on the interval $[0, 2\pi)$.
22. Solve $\tan \frac{x}{2} = \sqrt{3}$ on the interval $[0, 2\pi)$.

Trigonometric Functions

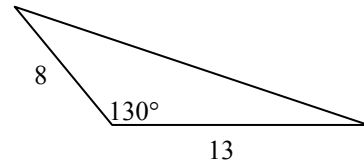
23. What is the domain of the function $f(x) = 8 \sin(7x) - 5$?
24. State the amplitude of the function $f(x) = 3 \sin(2x)$.
25. Find the period of the function $g(x) = 6 \cos(4x)$.
26. A function has a period of $3/2$. What is the frequency of the graph of the same function?
27. A sine wave has an equilibrium of $y = -1$ and an amplitude of $3/5$. What is the range of that curve?
28. What is the phase shift in the graph of $y = \sin(x + 6) - 5$?
29. State the vertical shift in the graph of $y = \cos(x - \pi) + 2$.
30. State the range of $y = 4 \sin 8x + 3$.
31. What is the period of the function $f(\alpha) = \tan(3\alpha)$?
32. Write an equation for the graph shown at the right.
33. From a calculator regression, a model sinusoid is found to be $y = 7.9 \sin(0.4x - 2.8) - 1.4$. What is the phase shift?
34. Sketch the graphs of $y = \tan x$, $y = \cot x$, $y = \sec x$, and $y = \csc x$.



Trigonometric Applications

35. Given $\triangle ABC$ where $\angle A = 25^\circ$, $\angle B = 68^\circ$, and $a = 18$, find b .

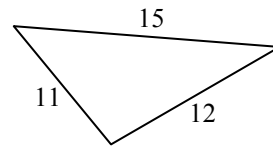
36. Find the missing side of the triangle shown at the right.



37. A triangle has sides that measure 4 feet, 8 feet, and 7 feet. Find the degree measure of the largest angle.

38. A triangle has two sides that measure 15 meters and 21 meters. The angle between these two sides is 98° . Find the area of the triangle.

39. Find the area of the triangle shown at the right.



40. Given $\triangle XYZ$ where $\angle X = 55^\circ$, $\angle Y = 87^\circ$, and $z = 3$, find x .

41. Suppose $\triangle ABC$ has the known measurements of $\angle A = 42^\circ$, $a = 22$, and $b = 12$. How many different (non-similar) triangles can be drawn that have these given dimensions?

42. For what cases should you initially use the Law of Sines or the Law of Cosines when solving triangles?

43. A triangular field needs fertilizer, which costs \$0.15 per square foot. The field has sides measuring 35 feet, 84 feet, and 91 feet. How much money is required to purchase the covering?

Vectors

44. Write the vector that has an initial point at $(4, -7)$ and a terminal point at $(-2, -11)$.

45. If $\mathbf{a} = \langle 2, -5 \rangle$ and $\mathbf{b} = \langle -8, 3 \rangle$, find $\mathbf{a} + \mathbf{b}$.

46. If $\mathbf{a} = \langle -6, 1 \rangle$ and $\mathbf{b} = \langle 7, -3 \rangle$, find $\mathbf{a} - \mathbf{b}$.

47. If $\mathbf{a} = \langle 8, -12 \rangle$, find $(3/2)\mathbf{a}$.

48. If $\mathbf{a} = \langle -6, -2 \rangle$, find $\|\mathbf{a}\|$.

49. A vector has a directional angle $\theta = -70^\circ$ and a magnitude of 5. Write the vector in component form.

50. A plane heads out on a bearing $N30^\circ E$ at a speed of 300 mph, but encounters wind blowing due east at a speed of 25 mph. Determine the resultant vector of the plane and the wind.

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|-----|------------------------|-----|---|-----|--|
| 1. | $a_n = 20 - 6n$ | 19. | $x = 5\pi/6, 7\pi/6$ | 36. | 19.149 |
| 2. | $a_n = 12(-3/2)^{n-1}$ | 20. | $x = 0, \pi$ | 37. | 88.977° |
| 3. | 21.5 | 21. | $x = 5\pi/8, 7\pi/8, 13\pi/8, 15\pi/8$ | 38. | 155.967 m ² |
| 4. | 3.815 | 22. | $x = 2\pi/3$ | 39. | 65.238 |
| 5. | 525 | 23. | $(-\infty, \infty)$ | 40. | 3.992 |
| 6. | 189 | 24. | 3 | 41. | one |
| 7. | 4 | 25. | $\pi/2$ | 42. | use Law of Sines for AAS, ASA, & SSA; use Law of Cosines for SAS & SSS |
| 8. | 2 | 26. | 2/3 | 43. | \$220.50 |
| 9. | -1/2 | 27. | $[-8/5, -2/5]$ | 44. | $\langle -6, -4 \rangle$ |
| 10. | 0 | 28. | left 6 | 45. | $\langle -6, -2 \rangle$ |
| 11. | 11π/12 | 29. | up 2 | 46. | $\langle -13, 4 \rangle$ |
| 12. | 15° | 30. | $[-1, 7]$ | 47. | $\langle 12, -18 \rangle$ |
| 13. | 1.343 | 31. | $\pi/3$ | 48. | $2\sqrt{10}$ |
| 14. | 29.209° | 32. | $y = 3 \sin(x/2)$ or $y = 3 \cos[1/2(x - \pi)]$ | 49. | $\langle 1.710, -4.698 \rangle$ |
| 15. | 4/3 | 33. | right 7 | 50. | $\langle 175, 259.808 \rangle$ |
| 16. | 5.498 | 34. | <i>see below</i> | | |
| 17. | 15.122° | 35. | 39.490 | | |
| 18. | 393 ft | | | | |

Graphs for #34:

