1st Semester Exam Review

The 1st Semester Exam will consist of 50 multiple-choice questions that relate to concepts presented throughout the course. 40% of the exam will address learning targets from the 1st Quarter, the remaining 60% will cover topics from the 2nd Quarter.

You will need a calculator on the semester exam, and there is a two hour time limit.

The questions that follow are similar in concept to those on the semester exam. You are encouraged to work through these before the exam in preparation (answers are attached at the end). You might also consider reviewing unit tests and quizzes. The 1st Quarter Exam makes a great review for the earlier topics, too.

Equations and Inequalities (20% of exam)

1. Solve $|x + 3| - 12 = 6$.
2. Solve $2x^2 + 4x = 3$.
3. Solve $x^3 + 2x^2 - 11x - 22 = 0$.
4. Solve $\sqrt{3x - 5} = 6$.
5. Solve $(4x - 5)^{3/2} = 27$.
6. Solve $e^{x+5} = 7$.
7. Solve $\log_2(x - 3) = 4$.
8. Solve $x^2 - 3x - 10 \le 0$.
9. Solve $14 + |x| < 8$.
10. Solve $\dfrac{5}{x + 4} \ge -1$.

Systems of Equations and Inequalities (20% of exam)

11. Describe the graphs for systems of linear equations that have no solution, one solution, and infinitely many solutions.

12. Let (x, y) be the solution to the system $\begin{cases} y = 2x + 4 \\ x - 2y = -3 \end{cases}$. Find the sum $x + y$.

13. Find the x-value in the solution for the system $\begin{cases} 2x - 5y = -1 \\ 3x + 4y = 2 \end{cases}$.

14. Solve the system $\begin{cases} y = \dfrac{1}{2}x + 5 \\ x - 2y = -15 \end{cases}$.

15. Solve the system $\begin{cases} y = x^2 + 4x \\ 5x - y = -2 \end{cases}$.

Systems of Equations and Inequalities (continued)

16. Solve $\begin{cases} x - y + z = 3 \\ \quad 2y - z = 1 \\ x + y - z = 7 \end{cases}$.

17. Find the *y*-value in the solution for the system $\begin{cases} z = 2x + y \\ y = 4 - z \\ x + y = 2 \end{cases}$.

18. Sketch the solution to the system $\begin{cases} 2x - y \le 0 \\ x - 2y > 4 \end{cases}$.

19. Sketch the solution to the system $\begin{cases} x \ge 1 \\ -2 < y < 3 \\ x - y \le 6 \end{cases}$.

20. Suppose the system of inequalities in #19 is the feasible region of an optimization problem. Maximize the objective function $f(x, y) = 4x + 3y$ over this feasible region.

Graph Behavior and Transformations (40% of exam)

21. Sketch the following parent functions:
    - quadratic
    - reciprocal
    - logarithmic
    - absolute value
    - exponential

22. If you are given the graph of $y = h(x)$, how could you sketch the graph of $y = 3\,h(2x)$?

23. Write the equation for the graph that results from a horizontal reflection over the *y*-axis of $y = \ln x$.

24. Describe the transformation from $y = x^2$ to $y = (x - 4)^2 - 1$.

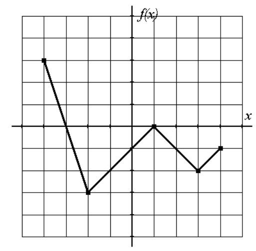
25. Describe the transformation from $y = |\,x\,|$ to $y = -|\,x/2\,|$.

26. Suppose the graph of $f(x) = x^3 - 3x^2 - 24x$. Find the following:
    - domain and range
    - relative maximums and minimums
    - end-behavior
    - intervals of increase and decrease

27. Suppose the graph of $y = \dfrac{3x - 1}{x + 2}$. Find the following:
    - domain and range
    - horizontal and vertical asymptotes
    - end-behavior
    - intervals of increase and decrease

28. Suppose the graph of $g(x) = \dfrac{x+6}{x^2-9}$. Find the following:

domain and range
end-behavior
horizontal and vertical asymptotes
intervals of increase and decrease



29. Suppose the function $y = f(x)$ whose graph is shown at the right.
Find the following:

domain and range
relative maximums and minimums
intervals of increase and decrease

30. Suppose the parabola defined by $y = 3x^2 - 12x + 1$. Find the following:

domain and range
vertex
end-behavior
vertex is maximum or minimum?
intervals of increase and decrease
concavity

31. Suppose the graph defined by $y = 2(3)^{-x}$. Find the following:

domain and range
asymptote
intervals of increase and decrease
end-behavior

32. Suppose the graph defined by $y = \log_3(x+4)$. Find the following:

domain and range
asymptote
intervals of increase and decrease
end-behavior

Modeling and Variation (20% of exam)

33. Suppose *P* varies directly with the square of *M* and inversely with *N*. Write a general variation model to represent this scenario?

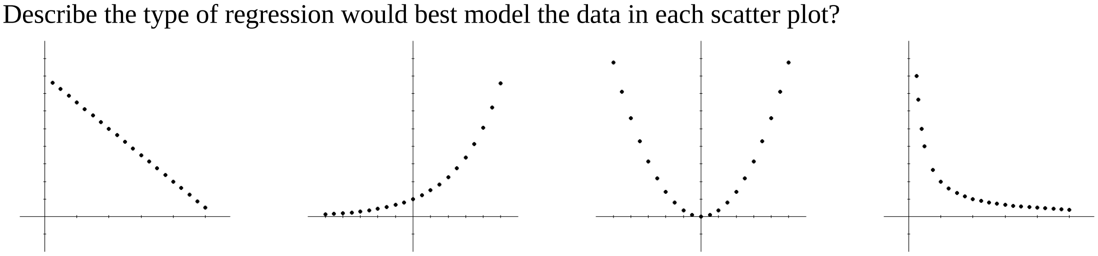
34. *A* is jointly proportional to *B* and *C*. It is known when *B* = 8 and *C* = 4, then *A* = 2. Write a specific variation model to represent the relationship between *A*, *B*, and *C*?

35. *y* varies directly with the square root of *x*. If *y* = 1 when *x* = 16, what is the constant of variation?

36. *M* and *N* vary inversely according to the table shown. Find the variation formula for the data?

M	1	2	3	4	5
N	120	60	40	30	24

37. Describe the type of regression would best model the data in each scatter plot?



For 38-41, use the data in the table at the right.

t	0	15	30	45	60
W	5	8	14	17	21

38. Find the equation for the function that bests model the data shown in the table?

39. Use the equation in #38 to predict the value of W when $t = 75$.

40. Use the equation in #38 to predict the value of t when $W = 30$.

41. Use the equation in #38 to predict the value of t when $W = 0$.

For 42-45, use the scenario below.

> The number of black bears in a state forest has dropped in recent years. This decrease has been modeled by the function $N(t) = 2.3(.94)^t$, where N is the number of bears (in hundreds) and t is the number of years since 2000.

42. According to the model function, what was the bear population in 1995?

43. What is the bear population predicted to be in 2010?

44. When can it be predicted that the number of bears will drop to 100?

45. It is assumed that the bears will become extinct if there is no intervention before their population drops to 20. Use the model equation to determine the latest year in which a conservation group can implement such a plan.

1st Semester Exam Review

Answers to Problems

Below are the answers to the problems from the 1st Semester Exam Review. While some additional notes are provided here, full solutions are not given. If you need additional help solving one or more of these exercises, please ask before, during, or after class.

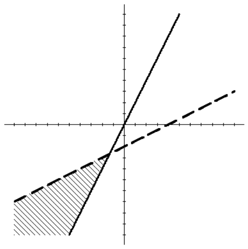
Equations and Inequalities (20% of exam)

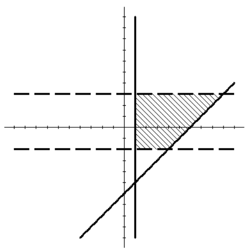
1. Solve graphically: $x = -21, 15$
2. Use the quadratic formula: $x = \frac{-2 \pm \sqrt{10}}{2}$
3. Find the first solution graphically, then use synthetic division to find the other two: $x = -2, \pm\sqrt{11}$
4. Square both sides, or solve graphically: $x = 41/3$
5. Raise both sides to the 2/3 power, or solve graphically: $x = 7/2$
6. Take the natural logarithm of both sides: $x = \ln 7 - 5$
7. Exponential both sides with a base of 2: $x = 19$
8. Solve graphically – look for where the graph is below the x-axis: $[-2, 5]$
9. Solve graphically – look for where the graph is below the x-axis: *no solution*
10. Solve graphically – look for where the graph is above the x-axis: $(-\infty, -9] \cup (-4, \infty)$

Systems of Equations and Inequalities (20% of exam)

11. A system with no solution will have parallel lines; a system with one solution will have intersecting lines; a system with infinitely many solutions will have coinciding lines (the lines are the same).
12. The solution is $(-5/3, 2/3)$, so $x + y = -1$.
13. The solution is $(6/23, 7/23)$, so $x = 6/23$.
14. *no solution*
15. Solve graphically to see two solutions: $(-1, -3)$ and $(2, 12)$
16. $(5, -1, -3)$
17. There are infinitely many solutions, so y could be any value.

Systems of Equations and Inequalities (continued)

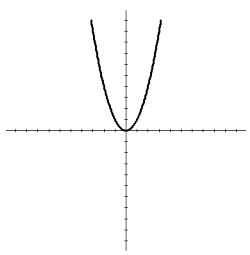
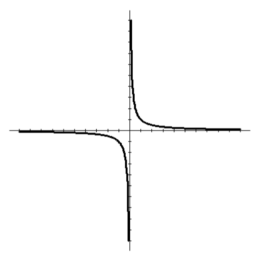
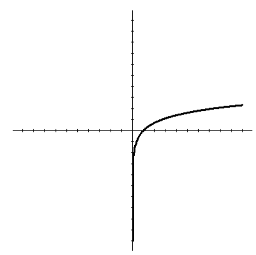
18. 

19. 

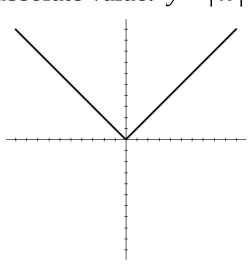
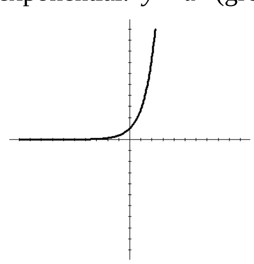
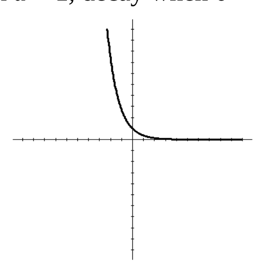
20. The vertices are (1, 3), (1, –2), (4, –2), and (9, 3). The maximum is $f(9, 3) = 45$.

Graph Behavior and Transformations (40% of exam)

21. quadratic: $y = x^2$ reciprocal: $y = 1/x$ logarithmic: $y = \log_b x$

absolute value: $y = |x|$ exponential: $y = a^x$ (growth when $a > 1$, decay when $0 < a < 1$)

22. Multiply the y-coordinates by 3 (for the vertical stretch) and divide the x-coordinates by 2 (for the horizontal shrink).

23. $y = \ln(-x)$

24. There is a horizontal shift right 4 units and a vertical shift down 1 unit.

Graph Behavior and Transformations (continued)

25. There is a horizontal stretch by a factor of 2 and a vertical reflection over the x-axis.

26. domain: $(-\infty, \infty)$
range: $(-\infty, \infty)$
rel. max.: $(-2, 28)$
rel. min.: $(4, -80)$
increase: $(-\infty, -2) \cup (4, \infty)$
decrease: $(-2, 4)$
left e.b.: $x \to -\infty,\ f(x) \to -\infty$
right e.b.: $x \to \infty,\ f(x) \to \infty$

27. domain: $(-\infty, -2) \cup (-2, \infty)$
range: $(-\infty, 3) \cup (3, \infty)$
horiz. asym.: $y = 3$
vert. asym.: $x = -2$
increase: $(-\infty, -2) \cup (-2, \infty)$
decrease: *none*
left e.b.: $x \to -\infty,\ y \to 3$
right e.b.: $x \to \infty,\ y \to 3$

28. domain: $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$
range: $(-\infty, -0.622] \cup (0, \infty)$
horiz. asym.: $y = 0$
vert. asym.: $x = -3,\ x = 3$
increase: $(-\infty, -3) \cup (-3, -0.804)$
decrease: $(-0.804, 3) \cup (3, \infty)$
left e.b.: $x \to -\infty,\ g(x) \to 0$
right e.b.: $x \to \infty,\ g(x) \to 0$

29. domain: $[-4, 4]$
range: $[-3, 3]$
rel. max.: $(1, 0)$
rel. min.: $(-2, -3),\ (3, -2)$
increase: $(-2, 1) \cup (3, 4)$
decrease: $(-4, -2) \cup (1, 3)$

30. domain: $(-\infty, \infty)$
range: $[-11, \infty)$
vertex: minimum at $(2, -11)$
concavity: upward
increase: $(2, \infty)$
decrease: $(-\infty, 2)$
left e.b.: $x \to -\infty,\ y \to \infty$
right e.b.: $x \to \infty,\ y \to \infty$

31. domain: $(-\infty, \infty)$
range: $(0, \infty)$
asymptote: $y = 0$
decrease: $(-\infty, \infty)$; never increasing
left e.b.: $x \to -\infty,\ y \to \infty$
right e.b.: $x \to \infty,\ y \to 0$

32. domain: $(-4, \infty)$
range: $(-\infty, \infty)$
asymptote: $x = -4$
increase: $(-4, \infty)$; never decreasing
right e.b.: $x \to \infty,\ y \to \infty$; no left e.b.

Modeling and Variation (20% of exam)

33. $P = \dfrac{kM^2}{N}$

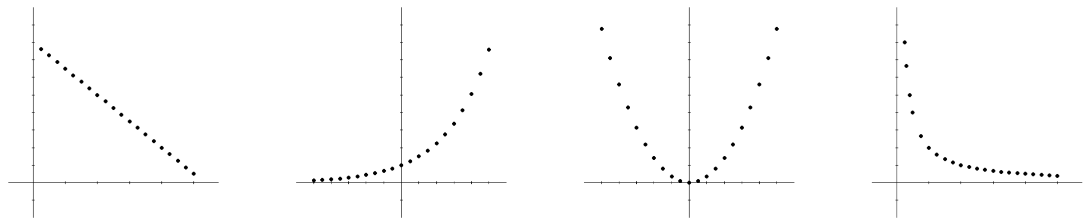
34. $A = \dfrac{1}{16} BC$

35. $k = 1/4$

Modeling and Variation (continued)

36. Either perform a power regression of use any set of points in the table: $N = \dfrac{120}{M}$

37. linear | exponential | quadratic | power (reciprocal)



38. Linear is great ($W = 0.273t + 4.8$), but quadratic is slightly better: $W = -0.000317t^2 + 0.292t + 4.657$

39. $W = 24.8$ ($W = 25.3$ if linear was used)

40. $t = 96.865$ ($t = 92.195$ if linear used)

41. $W = -15.662$ ($W = -17.561$ if linear used)

42. $N(-5) = 3.13$, so 313 bears.

43. $N(10) = 1.24$, so 124 bears.

44. $N = 1$ when $t = 13.461$, so in 2013.

45. $N = 0.2$ when $t = 39.472$, so in 2039.