Write a model equation for each scenario:

- *a* varies directly as *b*
- *y* varies inversely as *x*
- *m* varies jointly with *n* and *p*
- *t* varies as *s* and inversely as *r*
- *w* is proportional to *u* and *v*

Distance, *D*, varies directly as the square of time, *t*. If distance is measured in meters and time in seconds, answer the following:

- Write a general variation model to represent this situation.
- If $D = 12.25$ meters when $t = 5$ seconds, what is the constant of variation?
- Write the specific variation formula to represent this situation.
- Use the specific variation formula to find the distance when $t = 8$ seconds.
- Use the specific variation formula to find when the distance is 50 meters.

Income, *I*, is proportional to time, *T*, and inversely proportional to the square root of expense, *E*. Answer the following:

- Write a general variation model to represent this situation.
- If $I = 3$ when $T = 8$ and $E = 4$, what is the constant of variation?
- Write the specific variation formula to represent this situation.
- Use the specific variation formula to find the income when $T = 2$ and $E = 9$.
- Use the specific variation formula to find the time when $I = 0.5$ and $E = 16$.
- Use the specific variation formula to find the expense when $I = 5$ and $T = 12$.

t	1	2	3	4	5
$A(t)$	2	2	4	8	14
$B(t)$	26	20	16	13	10

Refer to the table above to answer the following.

- What type of model should be used to model $A(t)$. Why?
- Find the model equation for $A(t)$. Use appropriate variables.
- Find t when $A = 5$.
- Find A when $t = 7$.
- Find an exponential model for $B(t)$. Use appropriate variables.
- What is the value of B when $t = 0$?
- What is the decay rate for $B(t)$?
- Find t when $B = 6$.
- Find B when $t = 3.5$.

Solutions:

Write a model equation for each scenario:

- a varies directly as b
  - $a = kb$
- y varies inversely as x
  - $y = \frac{k}{x}$
- m varies jointly with n and p
  - $m=knp$
- t varies as s and inversely as r
  - $t = \frac{ks}{r}$
- w is proportional to u and v
  - $w = kuv$

Distance, D, varies directly as the square of time, t. If distance is measured in meters and time in seconds, answer the following:

- Write a general variation model to represent this situation.
  - $D = kt^2$
- If $D = 12.25$ meters when $t = 5$ seconds, what is the constant of variation?
  - $12.25 = k(5)^2 \quad \Rightarrow \quad k = 0.49$
- Write the specific variation formula to represent this situation.
  - $D = 0.49t^2$
- Use the specific variation formula to find the distance when $t = 8$ seconds.
  - $D = 0.49(8)^2 \quad \Rightarrow \quad D = 31.36$ meters
- Use the specific variation formula to find when the distance is 50 meters.
  - $50 = 0.49t^2 \quad \Rightarrow \quad 102.041 = t^2 \quad \Rightarrow \quad t = 10.102$ seconds (in this case, time cannot be negative so ignore the solution $t = -10.102$)

Income, I, is proportional to time, T, and inversely proportional to the square root of expense, E. Answer the following:

- Write a general variation model to represent this situation.
  - $I = \frac{kT}{\sqrt{E}}$
- If $I = 3$ when $T = 8$ and $E = 4$, what is the constant of variation?
  - $3 = \frac{k(8)}{\sqrt{4}} \quad \Rightarrow \quad k = 0.75$
- Write the specific variation formula to represent this situation.
  - $I = \frac{0.75T}{\sqrt{E}}$
- Use the specific variation formula to find the income when $T = 2$ and $E = 9$.
  - $I = \frac{0.75(2)}{\sqrt{9}} \quad \Rightarrow \quad I = 0.5$
- Use the specific variation formula to find the time when $I = 0.5$ and $E = 16$.
  - $0.5 = \frac{0.75T}{\sqrt{16}} \quad \Rightarrow \quad T = \frac{8}{3}$
- Use the specific variation formula to find the expense when $I = 5$ and $T = 12$.
  - $5 = \frac{0.75(12)}{\sqrt{E}} \quad \Rightarrow \quad \sqrt{E} = 1.8 \quad \Rightarrow \quad E = 3.24$

t	1	2	3	4	5
$A(t)$	2	2	4	8	14
$B(t)$	26	20	16	13	10

Refer to the table above to answer the following.

- What type of model should be used to model $A(t)$. Why?
  - Quadratic, since $R^2 = 1$
- Find the model equation for $A(t)$. Use appropriate variables.
  - $A(t) = t^2 - 3t + 4$
- Find t when $A = 5$.
  - $5 = t^2 - 3t + 4 \Rightarrow t = -0.303,\ 3.303$
- Find A when $t = 7$.
  - $A = (7)^2 - 3(7) + 4 \Rightarrow A = 32$
- Find an exponential model for $B(t)$. Use appropriate variables.
  - $B(t) = 32.503(0.791)^t$
- What is the value of B when $t = 0$?
  - $B(t) = 32.503(0.791)^0 \Rightarrow B = 32.503$
- What is the decay rate for $B(t)$?
  - The base is $1 - r = 0.791$, so $r = 0.209 \approx 21\%$
- Find t when $B = 6$.
  - $6 = 32.503(0.791)^t \Rightarrow t = 7.215$
- Find B when $t = 3.5$.
  - $B(t) = 32.503(0.791)^{3.5} \Rightarrow B = 14.321$