

Solving Linear Equations – A Graphical Approach

Introduction

A **linear equation in one variable** is any equation that can be written in the form

$$ax + b = 0.$$

The best part is, the **solution** to such an equation is always $x = -b/a$!

Unfortunately, most linear equations are not written in this simplified form which is why students spend a good portion of their algebra career learning various techniques for writing something like

$$\frac{2}{3}(5x - 7) = 3x + \left(\frac{1}{2} - x\right)$$

as $8x - 31 = 0$, only to get the relatively simple solution $x = 31/8$.

The purpose of this document is not to revisit those algebraic techniques – a review of these can be found online or in any algebra textbook – but rather to streamline the process into a single graphical approach.

The Technique

While there are several ways to solve linear equations **graphically**, this approach is usually the most efficient (and can be extended to non-linear equations as well):

1. **Set the equation equal to zero** by bringing everything on the right side to the left.
 - It might help to use parentheses around each side of the equation and move the right side as one big term. In other words, $(left\ side) - (right\ side) = 0$.
 - Don't worry about simplifying, although it doesn't hurt to do so.
2. **Graph the left side of the equation** (the non-zero part) using your calculator and find the x -intercept.
 - You might have to adjust your window settings to find the x -intercept.
 - There usually is only one x -intercept, but there are exceptions to this rule. See the notes below about these special cases.
3. Use the calculator's "zero" command to **find the value of the x -intercept**. This value is the solution to the original equation.
 - Some calculators will also rewrite a decimal value as a fraction

Special Cases

You might recall that while most linear equations have a single solution, it is also possible to have **no solution** (no value for x will make the equation true) or **infinitely many solutions** (any real number will make the equation true). You can see these scenarios graphically, too.

If the graph of the equation is a **horizontal line anywhere but on the x -axis**, there will be no solution.

- There is no solution because the horizontal line will not have an x -intercept.
- Using the “trace” command on the calculator will verify that the line is horizontal and not just one with a slight slope (in which case there would be an x -intercept eventually).

If the graph of the equation is a **horizontal line on the x -axis**, there are infinitely many solutions.

- Any real number will work because the horizontal line “intersects” the x -axis infinitely many times.
- Sometimes the line hides in the graph and you cannot see it on the x -axis. The “trace” command will bring up the cursor on the line which will help you see it better.

Final Thought

Before we look at a few examples on the next page, it should be noted that this technique does have its drawbacks. For one, exact solutions are only possible when the answers are rational values. If the solution was $x = \sqrt{2}$, all the calculator would give is $x = 1.4142135624$.

Of course, the variable in a linear equation is not always x . Unfortunately, most calculators cannot graph with any other letter, so you might have to use x instead of the actual variable.

One positive note: this technique works with non-linear equations as well. For example, you could solve a quadratic equation by setting it equal to zero and graphing the non-zero side. The result would be a parabola and the (real) solutions would be the x -intercepts.

Some Examples

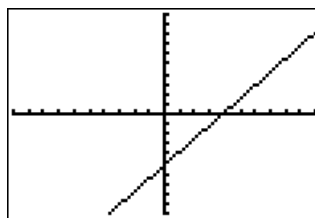
Example 1: Solve the equation $\frac{2}{3}(5x-7) = 3x + \left(\frac{1}{2} - x\right)$ graphically.

First, rewrite the equation as $\left[\frac{2}{3}(5x-7)\right] - \left[3x + \left(\frac{1}{2} - x\right)\right] = 0$.

Enter the left side of the equation into the calculator (watch the parentheses) and graph.

```

Plot1 Plot2 Plot3
\Y1=(2/3*(5X-7))
\Y2=-
\Y3=(3X+(1/2-X))
\Y4=
\Y5=
\Y6=
    
```



Now use the “zero” command to find the x -intercept.

```

MODE
1:value
2:zero
3:minimum
4:maximum
5:intersect
6:dy/dx
7:∫f(x)dx
    
```

```

Y1=(2/3*(5X-7))-(3X+(1/2-
Guess?
X=3.8297872 Y=-.0602837
    
```

```

Zero
X=3.875 Y=0
    
```

If needed, convert the decimal answer to a fraction (this can be done from the Home screen) to see that the solution is $x = 31/8$.

```

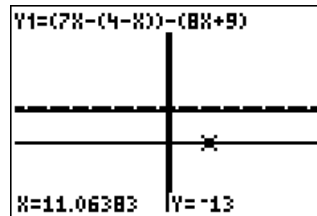
X>Frac
31/8
    
```

Example 2: Solve the equation $7x - (4 - x) = 8x + 9$ graphically.

Rewrite the equation as $[7x - (4 - x)] - [8x + 9] = 0$.

Enter the left side of the equation into the calculator and graph (the window shown was zoomed out once). Trace to verify the line is horizontal (notice the y-value is always -13).

```
Plot1 Plot2 Plot3
Y1=(7X-(4-X))-(
8X+9)
Y2=
Y3=
Y4=
Y5=
Y6=
```



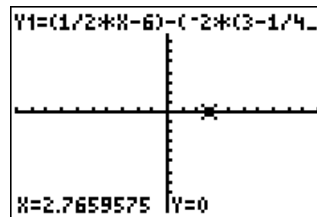
This equation has *no solution*.

Example 3: Solve the equation $\frac{1}{2}x - 6 = -2\left(3 - \frac{1}{4}x\right)$ graphically.

Rewrite the equation as $\left[\frac{1}{2}x - 6\right] - \left[-2\left(3 - \frac{1}{4}x\right)\right] = 0$.

Enter the left side of the equation into the calculator and graph. Trace to verify the line is horizontal and on the x -axis (notice the y -value is always 0).

```
Plot1 Plot2 Plot3
Y1=(1/2*X-6)-(-
2*(3-1/4*X))
Y2=
Y3=
Y4=
Y5=
Y6=
```



This equation has *infinitely many solutions* (in this case, the solution could be *any real number*).