A Summary of Graphical Analysis

The details below, while abbreviated, should help the typical algebra student in analyzing the graphs of common functions.

Domain

The domain is the set of x-values that can be used in the function to get an output. The domain is often stated as an interval of these x-values.

Graphically, the domain can be found moving along the x-axis (from left to right) and looking up or down. If the curve is above or below that point on the x-axis, that x-value is part of the domain.

Algebraically, look for functions that have denominators or square roots. Exclude x-values that make denominators zero or radicands negative from the domain.

Range

The range is the set of y-values that can be outputs for the function. The range is often stated as an interval of these y-values.

Graphically, the range can be found moving along the y-axis (from down to up) and looking left or right. If the curve is to either side of that point on the y-axis, that y-value is part of the range. Notice this is the same process as with domain, only your moving vertically and answering with y-values.

Intercepts

The x-intercept is where the function's graph touches the x-axis. It can be found by either setting the function equal to zero and solving or using the "zero" function in the calculator. It is best to write all x-intercepts as coordinates, $(a, 0)$.

The y-intercept is where the function's graph touches the y-axis. It can be found by either evaluating the function at $x = 0$ or using the "trace" feature on the calculator (when tracing, enter $x = 0$ and find the corresponding y-value). It is best to write y-intercepts as coordinates, $(0, b)$.

Relative Extreme Values

A relative maximum is the greatest function value on the graph in a small area. Think of it as the point at the top of a little “hill” in the graph. It does not have to be the greatest value for the entire function, just when compared to domain values to the immediate left and right.

A relative minimum is the least function value on the graph in a small area. Think of it as the point at the bottom of a little “valley” in the graph. It does not have to be the least value for the entire function, just when compared to domain values to the immediate left and right.

To find relative extreme values, use the “maximum” or “minimum” commands in the graphing calculator. The results can be stated as coordinates, (a, b), or with function notation, $f(a) = b$.

Note that endpoints cannot be considered for relative extreme values since there are no function values on both sides.

Intervals of Increasing and Decreasing Behavior

A graph is increasing if it has a positive slope (it is going “uphill”). A graph is decreasing if it has a negative slope (it is going “downhill”). The area where a graph is increasing or decreasing is described using x-values in interval notation (similar to domain).

To find the intervals, start at the left of the graph and determine if it is increasing or decreasing. Move to the right until the graph changes its direction or there is a jump in the graph. State the interval from the beginning to the x-value where the change occurred. Repeat this technique until another change happens. Continue the process until you reach the end of the graph.

Note that if a graph is neither increasing nor decreasing, it will be constant. While this doesn’t occur frequently, you would state the constant behavior using interval notation as well.

Asymptotes

A vertical asymptote is where a graph gets almost vertical and goes up or down in the y-direction forever. Where this occurs in the graph, the vertical asymptote is identified as a linear equation, $x = a$.

A horizontal asymptote is where a graph gets almost horizontal and goes left or right in the x-direction forever. Where this occurs in the graph, the horizontal asymptote is identified as a linear equation, $y = b$.