Below are sample solutions to the Honors Algebra 2 CCA Practice Exam (aka, the 2nd Semester Exam Review Packet).

1. In standard form, the slope is $m = \frac{-b}{a}$, so $m = \frac{-1}{-4} = \frac{1}{4}$. When $x = 0$, $y = -2$, so the y-intercept is (0, –2).

   Another option is to solve for slope-intercept form: $y = \frac{1}{4}x - 2$.

2. $4\left(4^2+1\right)-5\cdot 3^2 \quad \Rightarrow \quad 4(17)-5(9) \quad \Rightarrow \quad 68-45 \quad \Rightarrow \quad 23$

3. "a number decreased by six" is $(x - 6)$ and "the same number increased by three" is $(x + 3)$. Therefore, "the product is eight" means $(x - 6)(x + 3) = 8$.

4. Distribute (or FOIL) to see that $(2x + 1)(2x - 3)$ is the same as $4x^2 - 4x - 3$.

5. Use the "det" command on the calculator or the diagonal shortcut to find the determinant:

$$\begin{vmatrix} 2 & 3 & 1 \\ 0 & 1 & -2 \\ 1 & 2 & 3 \end{vmatrix} \begin{matrix} 2 & 3 \\ 0 & 1 \\ 1 & 2 \end{matrix} = \left[(2)(1)(3)+(3)(-2)(1)+(1)(0)(2)\right]-\left[(1)(1)(1)+(2)(-2)(2)+(3)(0)(3)\right]$$

$$= \left[6-6+0\right]-\left[1-8+0\right] = 7$$

6. $(4a)\left(3a^2w\right)\left(3aw^4\right)+\left(4a^2\right)\left(aw^3\right)^2 \quad \Rightarrow \quad 36a^4w^5+\left(4a^2\right)\left(a^2w^6\right) \quad \Rightarrow \quad 36a^4w^5+4a^4w^6$

7. Since the y-column for the determinant in the numerator is replaced with the constant column, the expression represents the y-value of the solution (by Cramer's rule).

   Note: There is a typo in this problem… the last equation in the system should read $x - 3y - 4z = -2$.

8. $\dfrac{x^2\left(a^{-3}y\right)^{-2}}{x^5a^3y^{-3}} \quad \Rightarrow \quad \dfrac{x^2a^6y^{-2}}{x^5a^3y^{-3}} \quad \Rightarrow \quad \dfrac{x^2a^6y^3}{x^5a^3y^2} \quad \Rightarrow \quad \dfrac{a^3y}{x^3}$

9. This is true by the zero product property where $a \cdot b = 0$, then $a = 0$ or $b = 0$.

10. This statement is false. Check the solutions to get $-4 \neq 1$ and $6 \neq 1$, respectively.

11. $3x^3+6x^2-9x=0 \quad \Rightarrow \quad 3x\left(x^2+2x-3\right)=0 \quad \Rightarrow \quad 3x(x+3)(x-1)=0$

$\Rightarrow \quad x=-3,\ 0,\ 1$

12. $\dfrac{x^2+4x}{x^2-6x+8}\cdot\dfrac{x^2-x-2}{3x^3+12x^2} \quad \Rightarrow \quad \dfrac{\cancel{x}\,\cancel{(x+4)}}{(x-4)\,\cancel{(x-2)}}\cdot\dfrac{\cancel{(x-2)}\,(x+1)}{3x^{\cancel{2}}\,\cancel{(x+4)}} \quad \Rightarrow \quad \dfrac{x+1}{3x(x-4)}$

$\Rightarrow \quad \dfrac{x+1}{3x^2-12x}$

13. $\dfrac{\dfrac{x^2+9x+20}{x^2-25}}{\dfrac{x+4}{x-4}} \quad \Rightarrow \quad \dfrac{\cancel{(x+4)}\,\cancel{(x+5)}}{(x-5)\,\cancel{(x+5)}}\cdot\dfrac{(x-4)}{\cancel{(x+4)}} \quad \Rightarrow \quad \dfrac{x-4}{x-5}$

14. Use the general form $y = a\,|\,x - c\,| + d$. "up 3 units" means $d = 3$, while "left 2 units" means $c = -2$. Make $|a| < 1$ to make the graph look wider (technically, this would be a vertical shrink but would look like a horizontal stretch). Finally, make $a < 0$ to cause a vertical reflection about the x-axis. $y = -2/5|\,x + 2\,| + 3$ meets these conditions.

15. $x^4-16 \quad \Rightarrow \quad \left(x^2+4\right)\left(x^2-4\right) \quad \Rightarrow \quad \left(x^2+4\right)(x+2)(x-2)$

16. $9x^2+36y^2 \quad \Rightarrow \quad 9\left(x^2+4y^2\right) \quad \Rightarrow \quad 9(x+2y)(x-2y)$

17. $\sqrt{12x^2y^3}\cdot\sqrt{24x^2y^4} \quad \Rightarrow \quad \sqrt{288x^4y^7} \quad \Rightarrow \quad 12x^2y^3\sqrt{2y}$

Note: The y^3 factor does not require absolute value bars since $y \geq 0$ from the first radical expression in the original problem.

18. $(2+3i)(5-i) \quad \Rightarrow \quad 10-2i+15i-3i^2 \quad \Rightarrow \quad 10-2i+15i-3(-1) \quad \Rightarrow \quad 13+13i$

19. One technique is to solve by factoring:

$$\begin{aligned} 4x^2+36&=0\\ x^2&=-9\\ x&=\pm\sqrt{-9}\\ x&=\pm 3i \end{aligned}$$

20. $i^{14} = i^4 \cdot i^4 \cdot i^4 \cdot i^2 = 1 \cdot 1 \cdot 1 \cdot (-1) = -1$

21. $\sqrt{-25}\cdot\sqrt{-12} \;\Rightarrow\; (5i)\cdot\left(2i\sqrt{3}\right) \;\Rightarrow\; 10i^2\sqrt{3} \;\Rightarrow\; -10\sqrt{3}$

22. $3\sqrt{24}-\sqrt{18}-\sqrt{96}+2\sqrt{50} \;\Rightarrow\; 3\cdot 2\sqrt{6}-3\sqrt{2}-4\sqrt{6}+2\cdot 5\sqrt{2} \;\Rightarrow\; 2\sqrt{6}+7\sqrt{2}$

23. Rewrite in standard form: $4x^2 + 8x + 3 = 0$. The discriminant is calculated as:

$$\begin{aligned} b^2-4ac &= 8^2-4(4)(3) \\ &= 64-48 \\ &= 16 \end{aligned}$$

24. Rewrite in standard form: $x^2 + 12x + 4 = 0$, then use the quadratic formula:

$$\begin{aligned} x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a} \\ &= \frac{-(12)\pm\sqrt{12^2-4(1)(4)}}{2(1)} \\ &= \frac{-12\pm\sqrt{128}}{2} \\ &= \frac{-12\pm 8\sqrt{2}}{2} \\ &= -6\pm 4\sqrt{2} \end{aligned}$$

25. Rewrite in standard form: $x^2 - 3x + 2 = 0$. The discriminant is calculated as:

$$\begin{aligned} b^2-4ac &= (-3)^2-4(1)(2) \\ &= 9-8 \\ &= 1 \end{aligned}$$

Since the discriminant is positive and a perfect square, there are two rational roots.

26. If x is the first odd integer, then $(x + 2)$ would be the next odd integer:

$$\begin{aligned} x(x+2)-\frac{1}{3}x &= 250 \\ 3x(x+2)-x &= 750 \\ 3x^2+5x-750 &= 0 \end{aligned}$$

27. The parent graph is shifted left 1 and down 4, so the boundary is $y = |x + 1| - 4$. Since the shaded region is above the boundary, the inequality is $y \geq |x + 1| - 4$.

28. The axis-of-symmetry is $x = \dfrac{-b}{2a} = \dfrac{-(16)}{2(-2)} = 4$.

29. The x-coordinate is $x = \frac{-b}{2a} = \frac{-(-12)}{2(-2)} = -3$, and the y-coordinate is $y(-3) = -4$.

30. $d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(4-3)^2 + (6+5)^2} = \sqrt{122}$

31. Complete the square to write in standard form:

$$x^2 + 4x + 4 + y^2 = 8 + 4$$

$$(x+2)^2 + y^2 = 12$$

The radius would be $\sqrt{12} = 2\sqrt{3}$.

32. Rewrite as $3x^2 + 6x + 2y^2 - 4y + 1 = 0$. Since the x^2 and y^2 terms are the same sign but have different coefficients, this represents an ellipse.

33. This is the geometric definition of a hyperbola.

34. Rewrite in standard form as $\frac{x^2}{36} + \frac{y^2}{9} = 1$, which has a horizontal major axis. The foci are a distance $\sqrt{36-9} = \sqrt{27} = 3\sqrt{3}$ to the left and right of the center, (0, 0).

35. The hyperbola opens up and down, so the vertices are a distance $\sqrt{25} = 5$ up and down from the center, (6, 4).

36. This is an ellipse that is centered at (–4, 2). The only possible choice that meets these conditions is $\frac{(x+4)^2}{16} + \frac{(y-2)^2}{4} = 1$.

37. Take the square root of both sides to get $\pm 3x = y$, which represents two lines that intersect at the origin (this is the degenerate case of the hyperbola).

38. The shaded region is inside the circle, so $x^2 + y^2 \le 9$. The shaded region is also to the left of the parabola, so $x \le -y^2$ or $y^2 + x \le 0$.

39. Use substitution where $y = (x + 1)^2$:

$$(x+1)^2 - \frac{y^2}{4} = 1$$
$$y - \frac{y^2}{4} = 1$$
$$y^2 - 4y + 4 = 0$$
$$(y-2)(y-2) = 0$$
$$y = 2$$

Substitute back in to find x when $y = 2$:

$$2 = (x+1)^2$$
$$\pm\sqrt{2} = x+1$$
$$x = -1 \pm \sqrt{2}$$

This gives the two coordinates as $\left(-1+\sqrt{2},\, 2\right)$ and $\left(-1-\sqrt{2},\, 2\right)$. Then the sum of all coordinates would be $\left(-1+\sqrt{2}\right)+\left(-1-\sqrt{2}\right)+\left(2\right)+\left(2\right) = 2$.

40. $p(-2) = 3(-2)^3 - 2(-2)^2 + a(-2) + b = -2a + b - 32$
$p(1) = 3(1)^3 - 2(1)^2 + a(1) + b = a + b + 1$
Then set up the system $\begin{cases} p(-2) = -37 \\ p(1) = 2 \end{cases} \Rightarrow \begin{cases} -2a + b - 32 = -37 \\ a + b + 1 = 2 \end{cases} \Rightarrow \begin{cases} -2a + b = -5 \\ a + b = 1 \end{cases}$,
which has the solution (2, –1). Therefore, $a = 2$.

41. By the Remainder Theorem, $r = 3(-1)^4 + 8(-1)^2 - 1 = 10$.

42. The “sideways” parabola as the x-axis as its axis-of-symmetry.

43. $g\left(f(x)\right) = g\left(x+1\right) = \left(x+1\right)^2 + 5\left(x+1\right) + 1 = x^2 + 2x + 1 + 5x + 5 + 1 = x^2 + 7x + 7$

44. From the graph, the possible y-values are anything from $y = 0$ to $y = 3$.

45. The speeds of the gears are 1/35 and 1/36, respectively. Set up a proportion to find the desired speed:

$$\frac{1/35}{1/56} = \frac{1450}{x} \quad \Rightarrow \quad x = 906.25$$

46. $z = kxy$. Use the givens to find k: $(12) = k(9)(4)$, so $k = 1/3$.
Now find z as $z = (1/3)(7)(2) = 14/3$.

47. $\dfrac{6(27)^{-1/3}}{3(27)^0 - 2(27)^{2/3}} \Rightarrow \dfrac{6(1/3)}{3(1) - 2(9)} \Rightarrow -\dfrac{2}{15}$

48. Graph the function to find the zeros (as x-intercepts): $x = -1/2, 1, 4$.

49. Since $2 + 3i$ is one root, so is its conjugate $2 - 3i$. Graph to find the last zero $x = -1$.

50. One technique is write both sides with a base of 3:

$$\begin{aligned} 9^{2x} &= (1/3)^{x-1} \\ \left(3^2\right)^{2x} &= \left(3^{-1}\right)^{x-1} \\ 3^{4x} &= 3^{-x+1} \\ 4x &= -x+1 \\ x &= 1/5 \end{aligned}$$

51. One technique is to rewrite in exponential form:

$$\log_3 x = -2 \quad \Rightarrow \quad x = 3^{-2} = \frac{1}{9}$$

52. Use properties to write as a single logarithmic expression: $\log_2\left(\dfrac{4^2 \cdot (1/2)}{8}\right) = \log_2(1) = 0$

53. $\log_x 15 = \log_x (3 \cdot 5) = \log_x 3 + \log_x 5 = 0.4777 + 0.699 = 1.1767$

54. One technique is to rewrite in logarithmic form, then use a calculator:

$$10^x = 3800 \quad \Rightarrow \quad x = \log_{10} 3800 = 3.58$$

55. Since every letter is unique, there are 8! arrangements.

56. Since A is duplicated, there are $\dfrac{7!}{2!} = 2520$ permutations.

57. $2 \cdot 3 \cdot 2 = 12$

58. Find the probability of “red then green” and “green then red”: $\dfrac{4}{15} \cdot \dfrac{5}{14} + \dfrac{5}{15} \cdot \dfrac{4}{14} = \dfrac{4}{21}$.

59. There are $C(20, 2)$ ways to form the group of two 7th-graders and $C(23, 2)$ ways to form the group of two 8th-graders. The total number of ways is $C(20, 2) \cdot C(23, 2) = 48{,}070$.

60. Perform a linear regression on the calculator to get the model equation $y = 0.97x + 1.13$.

61. Use the area formula $A = \pm\frac{1}{2}\begin{vmatrix} -4 & 7 & 1 \\ 5 & 3 & 1 \\ -1 & 1 & 1 \end{vmatrix} = -\frac{1}{2}(-42) = 21$.

62. A function and its inverse have graphs that are reflections over the line $y = x$. The only graphs that meet this condition are the parabolas.

63. This is an arithmetic sequence with a common difference of $d = 6$:

$$a_n = a_1 + d(n-1)$$
$$a_{75} = -21 + 6(75-1) = 423$$

64. First find the common ratio:

$$4 \cdot r^3 = -500$$
$$r^3 = -125$$
$$r = -5$$

Then the means are $4(-5) = -20$ and $4(-5)^2 = 100$.

65. $\sum_{n=3}^{6} 2^n = 8 + 16 + 32 + 64 = 120$

66. The distance on the fall is $15 + 10 + \frac{20}{3} + \frac{40}{9} + \cdots = \sum_{n=1}^{\infty} 15\left(\frac{2}{3}\right)^{n-1}$.

The distance on the bounce up is $10 + \frac{20}{3} + \frac{40}{9} + \cdots = \sum_{n=1}^{\infty} 10\left(\frac{2}{3}\right)^{n-1}$.

Add to get the total distance:

$$\sum_{n=1}^{\infty} 15\left(\frac{2}{3}\right)^{n-1} + \sum_{n=1}^{\infty} 10\left(\frac{2}{3}\right)^{n-1} = \frac{15}{1-(2/3)} + \frac{10}{1-(2/3)} = 45 + 30 = 75$$

67. This is an arithmetic sequence with a common difference of $d = 4$:

$$a_n = a_1 + d(n-1)$$
$$a_{100} = -20 + 4(100-1) = 376$$