The Problem

42. **PACKAGING** The Cookie Factory's best selling items are chocolate chip cookies and peanut butter cookies. They want to sell both types of cookies together in combination packages. The different-sized packages will contain between 6 and 12 cookies, inclusively. At least three of each type of cookie should be in each package. The cost of making a chocolate chip cookie is 19¢, and the selling price is 44¢ each. The cost of making a peanut butter cookie is 13¢, and the selling price is 39¢. How many of each type of cookie should be in each package to maximize the profit?

The Objective Function

The key to this problem is to realize that the objective is to maximize the *profit* and the information given is *expense* (or *cost*) and *income* (or *selling price*). Recall the relationship *profit* = *income* – *expense*.

Since the number of each type of cookie is needed to calculate the profit, those are the variables. Let C represent the number of chocolate chip cookies and P represent the number of peanut butter cookies.

The profit for each chocolate cookie would be 44¢ – 19¢ = 25¢. Similarly, the profit for each peanut butter cookie would be 39¢ – 13¢ = 26¢.

This information now allows us to write an objective function:

$$f(C, P) = 25C + 26P$$

The Constraints

The most obvious restriction on the variables is that each bag must contain at least three of each type of cookie. This can be translated as $C \geq 3$ and also $P \geq 3$.

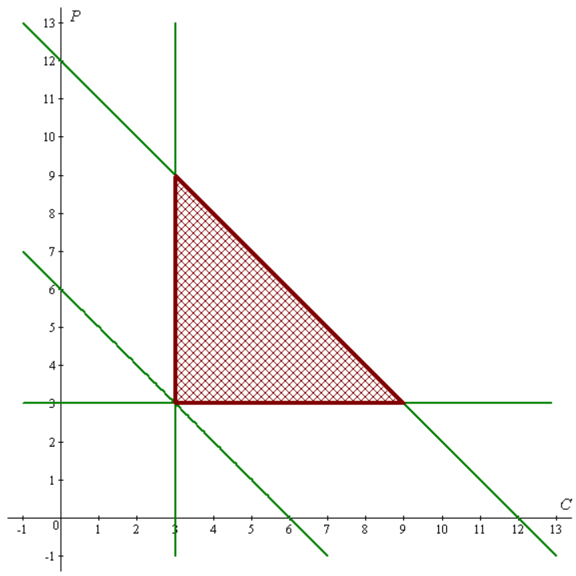
Since the packages of cookies can vary in size from six to twelve, this leads to the conclusion that $6 \leq C + P \leq 12$. This expression is perhaps easier to interpret (and graph) if written separately as $C + P \leq 12$ and $C + P \geq 6$.

As there are no other restrictions on the number of cookies, we have all of the constraints and can list them as a system of inequalities:

$$\begin{cases} C \geq 3 \\ P \geq 3 \\ C + P \geq 6 \\ C + P \leq 12 \end{cases}$$

The Feasible Region

Graph the constraints to find the feasible region.



The result is a bounded region shaped like a triangle (notice that the constraint $C + P \geq 6$ did not contribute to the results). The vertices of the feasible region are:

(3, 3), (9, 3), and (3, 9)

The Optimization

Finally, we can evaluate the objective function at each of these vertices to identify the maximum profit.

Vertex	*Function Evaluation*	*Function Value*
(3, 3)	$f(3, 3) = 25(3) + 26(3)$	153
(9, 3)	$f(9, 3) = 25(9) + 26(3)$	303
(3, 9)	$f(3, 9) = 25(3) + 26(9)$	309

From this table, we can conclude that packaging three chocolate chip cookies and nine peanut butter cookies results in the greatest profit. That profit would be $3.09 per package sold.