HONORS ALGEBRA 2
Chapter 10
Test A – Spring, 2009

Name ____________________
Date ____________________
Period ____________________

The solutions are in blue for each problem on the test. Calculators were allowed.

Solve each equation. State answers as exact values, <u>not</u> decimal approximations.

1. $6^{x-2} = 11$

$$\ln\left(6^{x-2}\right) = \ln 11$$
$$(x-2)\ln 6 = \ln 11$$
$$x-2 = \frac{\ln 11}{\ln 6}$$
$$x = \frac{\ln 11}{\ln 6} + 2$$

2. $\left(\frac{1}{5}\right)^{2x-3} = 125^{2-x}$

$$\left(5^{-1}\right)^{2x-3} = \left(5^{3}\right)^{2-x}$$
$$5^{-2x+3} = 5^{6-3x}$$
$$-2x+3 = 6-3x$$
$$x = 3$$

3. $7 - 4e^{2x} = 2$

$$-4e^{2x} = -5$$
$$e^{2x} = \frac{5}{4}$$
$$\ln\left(e^{2x}\right) = \ln\left(\frac{5}{4}\right)$$
$$2x = \ln\left(\frac{5}{4}\right)$$
$$x = \frac{1}{2}\ln\left(\frac{5}{4}\right)$$

4. $-3(2)^{1-4x} = 5$

$$2^{1-4x} = -\frac{5}{3}$$
$$\ln\left(2^{1-4x}\right) = \ln\left(-\frac{5}{3}\right)$$

no solution

5. $\log_4(x + 3) = 2$

$$4^{\log_4(x+3)} = 4^2$$
$$x+3=16$$
$$x=13$$

6. $\log_2 x + \log_2(x - 7) = 3$

$$\log_2\left(x^2-7x\right)=3$$
$$2^{\log_2\left(x^2-7x\right)}=2^3$$
$$x^2-7x-8=0$$
$$(x+1)(x-8)=0$$
$$x=-1,\,8$$
$$x=8$$

7. $5 + 3\ln(2x) = 17$

$$3\ln(2x)=12$$
$$\ln(2x)=4$$
$$e^{\ln(2x)}=e^4$$
$$2x=e^4$$
$$x=\frac{e^4}{2}$$

8. $\log(x + 8) - \log x = \log 3$

$$\log\left(\frac{x+8}{x}\right)=\log 3$$
$$\frac{x+8}{x}=3$$
$$x+8=3x$$
$$x=4$$

Answer the following.

9. The probability of having a car accident increases exponentially as the concentration of alcohol in the blood increases. This probability, P, is modeled by $P(x) = 0.06e^{12.77x}$, where x is the blood alcohol concentration (BAC).

a. What is the probability of someone having an accident when his/her BAC is 0.08?

$P(0.08) = 0.06e^{12.77(0.08)} = 0.167$ or 16.7%

b. What BAC practically guarantees an accident (that is, when is its probability 100%)?

$$1=0.06e^{12.77x} \quad\Rightarrow\quad \frac{1}{0.06}=e^{12.77x} \quad\Rightarrow\quad \ln\left(\frac{1}{0.06}\right)=\ln\left(e^{12.77x}\right)$$
$$\Rightarrow\quad \ln\left(\frac{1}{0.06}\right)=12.77x \quad\Rightarrow\quad x=\frac{\ln\left(\frac{1}{0.06}\right)}{12.77} \quad\Rightarrow\quad x=0.22$$

10. One bank offers a savings account at 3.4% annual percentage rate that is compounded every quarter. A competitor offers a continuously compounded savings account at 3.2%. Suppose you wanted to invest \$1200 for three years. Which bank should you choose? Explain your reasoning.

$$A_1(3) = 1200\left(1 + \frac{0.034}{4}\right)^{4(3)} = 1328.29 \qquad A_2(3) = 1200e^{0.032(3)} = 1320.91$$

The first bank would earn about \$7.38 more than the competitor.

11. Suppose the amount of a radioactive material decays according to the function $A(t) = A_0(0.9976)^t$, where A is measured in grams, t is measured in years, and A_0 represents the initial amount of the material.

a. State the decay rate of the radioactive material.

$$1 - r = 0.9976 \quad \Rightarrow \quad r = 0.0024 = 0.24\%$$

b. How long will it take a 200-gram sample to decay to 100 grams?

$$100 = 200(0.9976)^t \quad \Rightarrow \quad 0.5 = (0.9976)^t \quad \Rightarrow \quad \ln(0.5) = \ln\left(0.9976^t\right)$$

$$\Rightarrow \quad \ln(0.5) = t\ln(0.9976) \quad \Rightarrow \quad t = \frac{\ln(0.5)}{\ln(0.9976)} = 288.465$$

It will take about 288 years for the sample to decay to 100 grams.

12. Suppose the number of cells in an experiment grows exponentially. At $t = 0$, the number of cells is 105. After one hour, the number of cells has grown to 135. Find the number of cells after two hours.

$$135 = 105e^{k(1)} \quad \Rightarrow \quad \frac{135}{105} = e^k \quad \Rightarrow \quad \ln\left(\frac{135}{105}\right) = \ln\left(e^k\right) \quad \Rightarrow \quad k = \ln\left(\frac{135}{105}\right) = 0.2513$$

$$A(2) = 105e^{0.2513(2)} = 173.571$$

There would be about 174 cells after two hours.