HONORS ALGEBRA 2 Name ____________________
Chapter 3 Date ____________________
Test – KEY Period ____________________

The solutions are in blue for each problem on the test. Calculators were allowed.

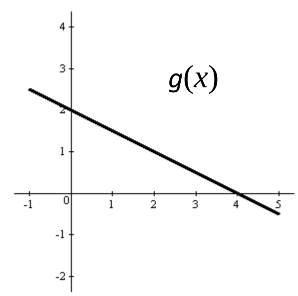
Assuming a system of linear equations in two variables, complete the following table.

If the graph of a system shows…	…and the system is described as…	…then the system has…
parallel lines	1. inconsistent	2. no solution
3. coinciding lines	consistent and dependent	4. infinitely many solutions
5. intersecting lines	6. consistent and independent	one solution

Answer the following.

7. Suppose $f(x) = 2x - 5$ and the graph of $g(x)$ is shown at the right. Solve the following system of equations.

$$\begin{cases} y = f(x) \\ y = g(x) \end{cases}$$



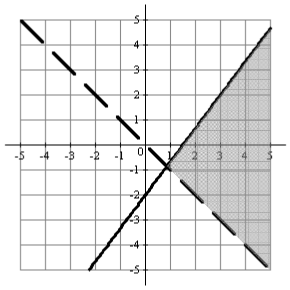
The equation of the line shown is $y = -\frac{1}{2}x + 2$, so the system can be solved using the substitution method.

$$2x - 5 = -\frac{1}{2}x + 2$$
$$\frac{5}{2}x = 7$$
$$x = \frac{14}{5}$$

$$y = 2\left(\frac{14}{5}\right) - 5$$
$$y = \frac{3}{5}$$

The solution is (14/5, 3/5).

8. Write the system of inequalities shown in the graph below.



$$\begin{cases} y > -x \\ y \le \frac{4}{3}x - 2 \end{cases}$$

Solve the following systems of equations.

9. $\begin{cases} 2x - 3y = 16 \\ x + \frac{1}{2}y = -2 \end{cases}$

Since the equations are in standard form, use elimination.

$$\begin{cases} 2x - 3y = 16 \\ \left(x + \frac{1}{2}y = -2\right) \times 6 \end{cases} \Rightarrow \begin{cases} 2x - 3y = 16 \\ 6x + 3y = -12 \end{cases} \Rightarrow \begin{array}{c} 8x = 4 \\ x = 1/2 \end{array}$$

$$\begin{cases} 2x - 3y = 16 \\ \left(x + \frac{1}{2}y = -2\right) \times -2 \end{cases} \Rightarrow \begin{cases} 2x - 3y = 16 \\ -2x - y = 4 \end{cases} \Rightarrow \begin{array}{c} -4y = 20 \\ y = -5 \end{array}$$

The solution is (1/2, –5).

10. $\begin{cases} y = \frac{2}{3}x - 1 \\ 4x = 6y + 6 \end{cases}$

Since *y* is already isolated in one equation, use substitution.

$$4x = 6\left(\frac{2}{3}x - 1\right) + 6$$

$$4x = 4x - 6 + 6$$

$$0 = 0$$

This is a true statement, so there are infinitely many solutions.

11. $$\begin{cases} x - y + z = -7 \\ 2x + y \quad = 1 \\ \quad 2y - z = 10 \end{cases}$$

Add the first and third equations to eliminate *z*.

$$\begin{cases} x - y + z = -7 \\ \quad 2y - z = 10 \end{cases} \Rightarrow \quad x + y = 3$$

Take this result and the second equation to find *x* and *y*.

$$\begin{cases} x + y = 3 \\ 2x + y = 1 \end{cases} \Rightarrow \begin{cases} (x + y = 3) \times -1 \\ 2x + y = 1 \end{cases} \Rightarrow \begin{cases} -x - y = -3 \\ 2x + y = 1 \end{cases} \Rightarrow \begin{matrix} x = -2 \\ y = 5 \end{matrix}$$

Substitute $x = -2$ and/or $y = 5$ into either the first or last equation to find $z = 0$. Therefore the solution is (–2, 5, 0).

12. $$\begin{cases} 2x + 3y - z = 0 \\ -4x - 6y + 2 = 0 \\ 2x + 3y - z = 5 \end{cases}$$

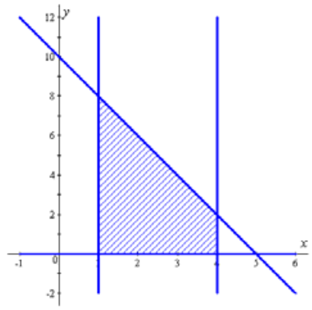
Subtract the third equation from the first one to get a false statement $0 = -5$. This means there is no solution to this system.

NOTE: If you start with the first and second equations, elimination results in a true statement. This requires that you check with the third equation to see if there are infinitely many or no solutions.

Use the given constraints to find the maximum and minimum values of the objective function.

13. $f(x, y) = 4x + 7y$

$$\begin{cases} 1 \le x \le 4 \\ y \ge 0 \\ 2x + y \le 10 \end{cases}$$



Vertices:
(1, 0), (4, 0)
(4, 2), (1, 8)

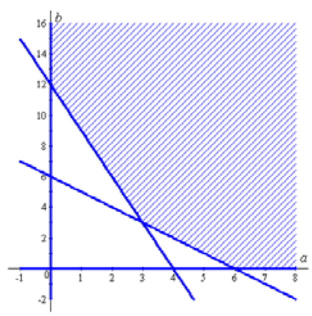
Use the vertices in the objective function:

$f(1, 0) = 4(1) + 7(0) = 4$
$f(4, 0) = 4(4) + 7(0) = 16$
$f(4, 2) = 4(4) + 7(2) = 30$
$f(1, 8) = 4(1) + 7(8) = 60$

The maximum value of the function is $f(1, 8) = 60$.
The minimum value of the function is $f(1, 0) = 4$.

14. $g(a, b) = a + 5b$

$$\begin{cases} a \ge 0, \quad b \ge 0 \\ b \ge 6 - a \\ b \ge 12 - 3a \end{cases}$$



Vertices:
(0, 12), (3, 3)
(6, 0)

Since the feasible region is unbounded, the objective function will not have both a maximum and a minimum. Use a point in the unbounded region to see if the values get larger or smaller – this solution will use the point (10, 15). Use the vertices and extra point in the objective function:

$g(0, 12) = (0) + 5(12) = 60$
$g(3, 3) = (3) + 5(3) = 18$
$g(6, 0) = (6) + 5(0) = 6$
$g(10, 15) = (10) + 5(15) = 85$

There is no maximum value for the function.
The minimum value of the function is $g(6, 0) = 6$.