HONORS ALGEBRA 2
Chapter 4
Test – KEY

Name ______________________
Date ______________________
Period ______________________

The solutions are in blue for each problem on the quiz. Calculators were allowed.

Evaluate the following expressions. Show all steps in the calculations.

1. $\begin{bmatrix} 1 & -2 \\ x & 0 \end{bmatrix} \cdot \begin{bmatrix} 4 & x & 0 \\ -1 & 2 & 3 \end{bmatrix}$

Since the dimensions are 2×2 and 2×3, the product is defined and will be 2×3.

$$\begin{bmatrix} (1)(4)+(-2)(-1) & (1)(x)+(-2)(2) & (1)(0)+(-2)(3) \\ (x)(4)+(0)(-1) & (x)(x)+(0)(2) & (x)(0)+(0)(3) \end{bmatrix} \Rightarrow \begin{bmatrix} 6 & (x-4) & -6 \\ 4x & x^2 & 0 \end{bmatrix}$$

2. $\begin{bmatrix} -3 & -1 \\ 2 & 5 \end{bmatrix}^{-1}$

First find the determinant of the matrix: $\begin{vmatrix} -3 & -1 \\ 2 & 5 \end{vmatrix} = (-3)(5)-(2)(-1) = -13$.

$$\begin{bmatrix} -3 & -1 \\ 2 & 5 \end{bmatrix}^{-1} \Rightarrow \frac{1}{-13}\begin{bmatrix} 5 & 1 \\ -2 & -3 \end{bmatrix} \Rightarrow \begin{bmatrix} -5/13 & -1/13 \\ 2/13 & 3/13 \end{bmatrix}$$

3. $\begin{vmatrix} 2 & -4 \\ a & 5 \end{vmatrix}$

$$\begin{vmatrix} 2 & -4 \\ a & 5 \end{vmatrix} = (2)(5)-(a)(-4) = 10+4a$$

4. $\begin{vmatrix} 0 & 3 & 2 \\ -1 & 5 & -4 \\ 1 & -2 & 0 \end{vmatrix}$

$$\left|\begin{array}{ccc|cc} 0 & 3 & 2 & 0 & 3 \\ -1 & 5 & -4 & -1 & 5 \\ 1 & -2 & 0 & 1 & -2 \end{array}\right.$$
$$=[(0)(5)(0)+(3)(-4)(1)+(2)(-1)(-2)]-[(1)(5)(2)+(-2)(-4)(0)+(0)(-1)(3)]$$
$$=[0-12+4]-[10+0+0]$$
$$=-18$$

Use matrices to solve the following system of equations.

5. $\begin{cases} x+y+z=3 \\ 2x+y+4z=8 \\ x+2y-z=1 \end{cases}$ $\quad rref\left(\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 2 & 1 & 4 & 8 \\ 1 & 2 & -1 & 1 \end{array}\right]\right) \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 3 & 5 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$

Since the last row is a true statement (and there are no false statements in the other two rows to contradict it), there are infinitely many solutions to the system.

6. $\begin{cases} x-3y+z=4 \\ -x+2y=3 \\ -x+3y-z=2 \end{cases}$ $\quad rref\left(\left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ -1 & 2 & 0 & 3 \\ -1 & 3 & -1 & 2 \end{array}\right]\right) \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

Since the last row is a false statement, there are no solutions to the system.

7. $\begin{cases} x-y+2z=-3 \\ 2x+y-z=0 \\ -x+2y-3z=7 \end{cases}$ $\quad rref\left(\left[\begin{array}{ccc|c} 1 & -1 & 2 & -3 \\ 2 & 1 & -1 & 0 \\ -1 & 2 & -3 & 7 \end{array}\right]\right) \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 3 \end{array}\right]$

The solution to the system is $(-2, 7, 3)$.

Answer the following.

8. Suppose $A = \begin{bmatrix} 3 & -1 \\ 5 & 2 \end{bmatrix}$ and $A \cdot X = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$. Solve for X.

One technique is to solve the equation using the inverse of A.

$$A \cdot X = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$$
$$X = A^{-1} \cdot \begin{bmatrix} -2 \\ 4 \end{bmatrix}$$
$$X = \begin{bmatrix} 3 & -1 \\ 5 & 2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} -2 \\ 4 \end{bmatrix}$$
$$X = \frac{1}{11}\begin{bmatrix} 2 & 1 \\ -5 & 3 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 4 \end{bmatrix}$$
$$X = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

9. Suppose M is a 3 × 3 matrix and $|M| \neq 0$. Evaluate $M \cdot M^{-1}$.

Since M is a square matrix with a non-zero determinant, it has an inverse. By definition, $M \cdot M^{-1} = I$, where I is the identity matrix.

Since M was 3 × 3, so is its identity. Therefore, $M \cdot M^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

10. Find the area of the triangle with vertices (–3, –4), (3, 0), and (6, –2).

$$A = \pm\frac{1}{2}\begin{vmatrix} -3 & -4 & 1 \\ 3 & 0 & 1 \\ 6 & -2 & 1 \end{vmatrix} = \pm\frac{1}{2}\left[(0-24-6)-(0+6-12)\right] = 12$$

11. A metal supply company has two silver alloys on hand. One is 22% silver while the another is 30% silver. How many of grams of each alloy is required to produce 80 grams of a new alloy that is 27% silver?

Let x represent the amount in grams of the 22% alloy and y represent that amount in grams of the 30% alloy.

One equation can be written to represent the total alloy amounts: $x + y = 80$.

Another equation can represent the total amount of silver: $0.22x + 0.30y = 0.27(80)$.

$$\begin{cases} x + y = 80 \\ 0.22x + 0.30y = 0.27(80) \end{cases} \Rightarrow \begin{array}{l} x = 30 \\ y = 50 \end{array}$$

30 grams of the 22% alloy and 50 grams of the 30% alloy are required to produce 80 grams of the 27% alloy.

12. The points (−1, 10), (0, 5), and (2, 14) are on a parabola. Find the equation of the parabola using the quadratic equation $y = ax^2 + bx + c$.

Substitute the coordinates into the quadratic equation to get three equations in terms of a, b, and c.

$$\begin{cases} a - b + c = 10 \\ c = 5 \\ 4a + 2b + c = 14 \end{cases} \Rightarrow \begin{array}{l} a = 19/6 \\ b = -11/6 \\ c = 5 \end{array}$$

The equation that contains the three points would be $y = \frac{19}{6}x^2 - \frac{11}{6}x + 5$.