.Question 1

1a) $\int_0^6 f(t)\,dt = 142.275 \text{ ft}^3$

1b) $f(8) - g(8) = 48.417 - 108 = -59.583 \text{ ft}^3\text{/hr}$

1c) $h(t) = \begin{cases} 0, & 0 \le t < 6 \\ 125(t-6), & 6 \le t < 7 \\ 125 + 108(t-7), & 7 \le t \le 9 \end{cases}$

1d) $\int_0^9 f(t)\,dt - h(9) = 367.335 - 341 = 26.335 \text{ ft}^3$

Question 2

2a) $E'(6) \approx \dfrac{E(7) - E(5)}{7-5} = \dfrac{21-13}{7-5} = 4$ (400 entries per hour)

2b) $\dfrac{1}{8}\int_0^8 E(t)\,dt \approx \dfrac{1}{8}\left[\dfrac{2}{2}(4-0) + \dfrac{3}{2}(13-4) + \dfrac{2}{2}(21-13) + \dfrac{1}{2}(23-21)\right] = 3.6875$

This means there was an average of approximately 368.75 entries added to the box each hour from noon to 8 PM.

2c) $23 - \int_8^{12} P(t)\,dt = 23 - 16 = 7,$ so 700 entries had not yet been processed

2d) $P'(t) = 0$ when $t = 9.18350$ and $t = 10.81650$

$P(8) = 0$; $P(9.18350) = 5.089$; $P(10.81650) = 2.911$; $P(12) = 8$

The maximum rate at which the entries are being processed occurs when $t = 12$, or midnight.

Question 3

3a) $$speed\Big|_{t=3} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\Bigg|_{t=3} = \sqrt{\left(2\cdot 3-4\right)^2 + \left(3e^0 - 1\right)^2} = 2\sqrt{2}\ \text{m/s}$$

3b) $$\int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\,dt = \int_0^4 \sqrt{\left(2t-4\right)^2 + \left(te^{t-3} - 1\right)^2}\,dt = 11.588\ \text{m}$$

3c) $$\frac{dy}{dx} = \frac{te^{t-3}-1}{2t-4} = 0 \quad \text{when } t = 2.20794$$

$$\frac{dx}{dt}\Bigg|_{t=2.20794} = 2\left(2.20794\right) - 4 = 0.416 > 0,$$ so the particle is moving right.

3d) i) $x(t) = 5$ when $t = 1$ and $t = 3$

ii) $$\frac{dy}{dx}\Bigg|_{t=1} = \frac{te^{t-3}-1}{2t-4}\Bigg|_{t=1} = 0.432 \quad \text{and} \quad \frac{dy}{dx}\Bigg|_{t=3} = \frac{te^{t-3}-1}{2t-4}\Bigg|_{t=3} = 1$$

iii) $$y(1) = y(2) + \int_2^1 \left(te^{t-3} - 1\right)dt = \left(3 + \frac{1}{e}\right) + 0.632 = 4 \quad \text{or}$$

$$y(3) = y(2) + \int_2^3 \left(te^{t-3} - 1\right)dt = \left(3 + \frac{1}{e}\right) + 0.632 = 4$$

Question 4

4a) $$\int_0^9 \left(6 - 2\sqrt{x}\right)dx = \left[6x - \frac{4}{3}x^{3/2}\right]_0^9 = 18 - 0 = 18 \quad \text{or} \quad \int_0^6 \left(\frac{y^2}{4}\right)dy = \left[\frac{y^3}{12}\right]_0^6 = 18 - 0 = 18$$

4b) $$\int_0^9 \left[\left(7 - 2\sqrt{x}\right)^2 - \left(7-6\right)^2\right]dx$$

4c) $$\int_0^6 \left(3\cdot\frac{y^4}{16}\right)dy$$

Question 5

5a) $y(0.5) \approx 0 + 1(-0.5) = -0.5$

$y(0) \approx -0.5 + 1.5(-0.5) = -1.25$

5b) $\lim_{x\to 1}\dfrac{f(x)}{x^3-1} = \lim_{x\to 1}\dfrac{f'(x)}{3x^2} = \dfrac{f'(1)}{3(1)^2} = \dfrac{1}{3}$

5c) $\int \dfrac{dy}{1-y} = \int dx \quad \Rightarrow \quad -\ln\left|1-y\right| = x + C_1 \quad \Rightarrow \quad y = 1 + Ce^{-x}$

$0 = 1 + Ce^{-1} \quad \Rightarrow \quad C = -e \quad \Rightarrow \quad y = 1 - e^{1-x}$

Question 6

6a) $\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} + \cdots + \dfrac{(-1)^n x^{2n}}{(2n)!} + \cdots$

$f(x) = -\dfrac{1}{2!} + \dfrac{x^2}{4!} - \dfrac{x^4}{6!} + \cdots + \dfrac{(-1)^{n+1} x^{2n}}{(2n+2)!} + \cdots$

6b) $f'(0) = 1\cdot c_1 = 0$ and $f''(0) = 2\cdot c_2 = 2\cdot\dfrac{1}{24} = \dfrac{1}{12} > 0$

Since $f'(0) = 0$ and $f''(0) > 0$, there is a relative minimum

6c) $g(x) \approx 1 - \dfrac{x}{2!} + \dfrac{x^3}{3\cdot 4!} - \dfrac{x^5}{5\cdot 6!}$

6d) $g(1) \approx 1 - \dfrac{1}{2!} + \dfrac{1^3}{3\cdot 4!} = 1 - \dfrac{1}{2} + \dfrac{1}{72} = \dfrac{37}{72}$

$\left|g(1) - \dfrac{37}{72}\right| \le \dfrac{(1)^5}{5\cdot 6!} < \dfrac{1}{6!}$, by the Alternating Series Estimation Theorem