AP Calculus BC
Summer Preparation Exercises

These exercises represent some of the more fundamental concepts needed upon entering AP Calculus BC. This "packet" is expected to be completed and brought to class on the first full day of school.

As you work through these problems, you most likely will come across topics that require a little review… you might even find some that you have completely forgotten! When this situation presents itself, get help. There are many resources available – websites, textbooks, previous courses' notes, etc. – that provide a plethora of information to assist you.

If you have any questions or need additional help during the summer, please email Mr. Deck at either kevind@wcs.edu or mrdeck@bellsouth.net.

General Algebraic Concepts

1. Let f be a linear function where $f(2) = -5$ and $f(-3) = 1$. State the function $f(x)$.

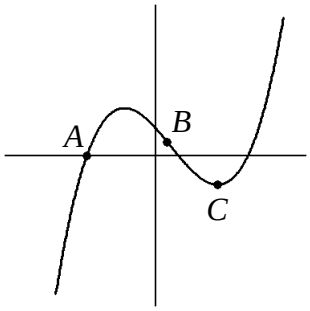
2. Find an equation for the line that contains the coordinate (5, 1) and is perpendicular to the line $6x - 3y = 2$.

3. Use the table of values at the right to calculate the average rate of change from $t = 1$ to $t = 4$.

t	0	1	2	3	4
$x(t)$	8	7	5	1	2

4. The cost for text messaging (c, in dollars) is dependent upon the number of messages sent (n). If the cost is modeled by the function $c(n) = 0.05n + 2.50$, what is the rate of change in the cost with respect to number of sent messages? State the units in your answer.

5. Three points – A, B, and C – are identified in the graph at the right. Order, from least to greatest, the rates of change at those three points.



6. Find the distance between the points (8, –1) and (–4, –6).

7. Rationalize the numerator in the expression $\dfrac{\sqrt{4+h}-2}{h}$.

8. Evaluate $\displaystyle\sum_{n=0}^{4}\frac{(n+1)^2}{n!}$.

9. Suppose $g(x) = \dfrac{x}{x+3}$. Find the inverse function of g, $g^{-1}(x)$.

10. If $(f \circ g)(x) = 8 + \sqrt{4-3x}$, state a possible function for both $f(x)$ and $g(x)$.

11. Find the points of intersection in the graphs of $y = x - 1$ and $y^2 = 2x + 6$.

12. Write the following expressions in simplified, factored form:
    - $3x^3 + 192$
    - $2x^3 - 11x^2 + 12x + 9$
    - $2x^{5/4} + x^{3/4} - 15x^{1/4}$
    - $x(1-2x)^{-3/2} + (1-2x)^{-1/2}$
    - $\left(\frac{2}{x} - 3\right) \div \left(1 - \frac{1}{x-1}\right)$

13. If $x > 3$, rewrite $\frac{1}{2}\ln(x-3) + \ln(x+2) - 6\ln x$ as a single logarithmic expression.

14. Rewrite the exponential expression 4^x with base e.

15. Rewrite $\sin(2x)$ in terms of a single angle, x.

16. Simplify the expression $2\sin^2 x + \cos(2x)$.

17. Rewrite the expression $\cos^2 x$ to reduce the exponent to 1.

18. Suppose $\mathbf{a} = \langle 5, -12 \rangle$. Find the magnitude of $\mathbf{a}$, $|\mathbf{a}|$.

19. If $a_n = 64, -32, 16, -8, \ldots$, evaluate $\sum_{i=1}^{\infty} a_n$.

20. Evaluate the following without a calculator:
    - $\sin(7\pi/6)$
    - $\cos(120°)$
    - $\tan(\pi/2)$
    - $\csc(60°)$
    - $\sec(-2\pi/3)$
    - $\cot(-135°)$

Equations and Inequalities

21. Solve each of the following equations for all real values of x:
    - $\frac{3}{7}(x+5) - 2x = 13 - \frac{4x}{3}$
    - $3x^2 + 2x = 6$
    - $x^3 - x^2 - 8x + 12 = 0$
    - $4e^{2x} = 5$
    - $\ln(3x)^2 = 16$
    - $3\sqrt{x-2} - 8 = 8$
    - $(x-4) - 5(x-4)^{1/2} = 6$
    - $\cos(3x) = 0$
    - $2\sin^2 x = \sin x + 1$
    - $\frac{3}{x-1} - \frac{8}{9} = \frac{1}{x+5}$
    - $|6 - 7x| = x + 4$

22. Solve each of the following inequalities for all real values of x:
    - $x^2 > 2x + 8$
    - $\left|2 - \frac{x}{3}\right| < 5$
    - $\frac{x-5}{3-x} \geq 0$
    - $\sin x < \cos x$ on the interval $[0, 2\pi]$

Coordinate Geometry

23. Sketch each of the following functions without a calculator, then state its domain and range:

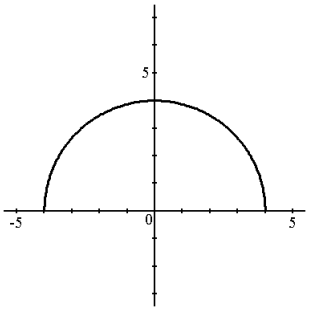
- $f(x) = (x-3)^2 + 2$
- $f(x) = \frac{1}{2}(x+1)^3$
- $f(x) = 2|x-4| - 3$
- $f(x) = 2 - \sqrt{x}$
- $f(x) = \sqrt[3]{1-x}$
- $f(x) = \frac{1}{x+1}$
- $f(x) = \frac{-3}{x^2}$
- $f(x) = 5\sin x$
- $f(x) = \cos(\pi x)$
- $f(x) = \tan\left(x - \frac{\pi}{4}\right)$
- $f(x) = e^{-x}$
- $f(x) = \ln(x+2)$

24. Sketch a graph of the piecewise function $f(x) = \begin{cases} x^2 - 5, & x < -1 \\ 0, & x = -1 \\ 6 - 4x, & x > -1 \end{cases}$.

25. Identify the vertical and horizontal asymptotes in the graph of $y = \frac{3x^2 + 5}{4 - x^2}$.

25. Describe the end-behavior of the function $f(x) = -3^x$.

26. State the function for the semicircle shown at the right.



27. Sketch a graph of the parabola defined by $x = (y+3)(y-1)$.

28. Sketch a graph of the hyperbola defined by $x^2 - y^2 = 4$.

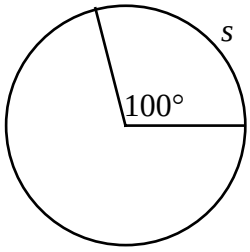
29. Sketch a curve defined by the parametric equations $x = t^2$ and $y = t + 1$ on the interval $-2 \le t \le 3$.

30. Rewrite the polar coordinate $(-2, 5\pi/4)$ as a rectangular coordinate.

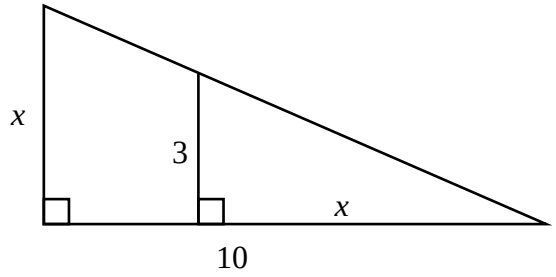
Planar and Spatial Geometry

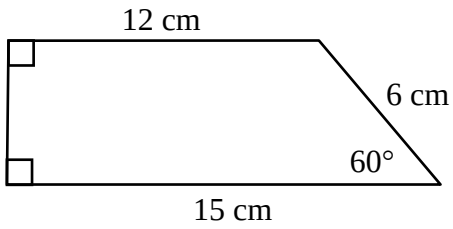
31. Use the circle at the right has a radius if 6 ft. Find the following:

- the area of the circle
- the circumference of the circle
- the length of arc s



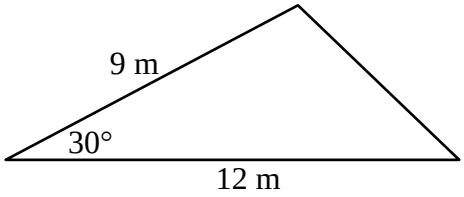
32. Find the value of x in the triangle shown below.





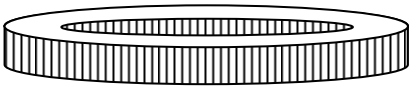
33. Find the area of the trapezoid shown above.

34. Use the triangle shown at the right to find the following:
    - the missing side of the triangle
    - the area of the triangle



35. Suppose a sphere has a radius of $r = 4$ inches. Find the following:
    - the circumference of the "great circle" to the sphere
    - the volume of the sphere
    - the surface area of the sphere

36. The figure below is the shape of a washer with an outer radius of 18 ft and an inner radius of 15 ft. If the figure is 3 ft in height, find the volume of the washer.



A Few Final Comments…

Students often ask what they should work on over the summer break to prepare for AP Calculus in the fall. While a list of prerequisite skills are defined by the College Board, the following are perhaps the most important:

- Possess a general understanding of graph behavior, including knowing the parent graphs of all major functions.
- Use algebra to manipulate mathematical expressions and equations, including exponentials and logarithms.
- Perform calculations involving right triangle trigonometry, including the special angles common in the unit circle.
- Display the graph of any function using a graphing calculator, adjusting the viewing window settings as needed.