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Suppose $f(2) = -g'(2) = 5$, $f'(2) = 4$, and $g(2) = -3$. Evaluate the following expressions, showing all steps that lead to your conclusion. (8 points each)

$$\frac{d}{dx}\left[5f\left(\sqrt{x}\right)\right]_{x=4} = \left[5f'\left(\sqrt{x}\right)\cdot\frac{1}{2\sqrt{x}}\right]_{x=4}$$

$$= 5f'\left(\sqrt{4}\right)\cdot\frac{1}{2\sqrt{4}}$$

$$= 5\cdot 4\cdot\frac{1}{4}$$

$$= 5$$

$$\frac{d}{dx}\left[f(2x)\cdot g(3x-1)\right]_{x=1} = \left[f'(2x)\cdot 2\cdot g(3x-1) + f(2x)\cdot g'(3x-1)\cdot 3\right]_{x=1}$$

$$= f'(2\cdot 1)\cdot 2\cdot g(3\cdot 1-1) + f(2\cdot 1)\cdot g'(3\cdot 1-1)\cdot 3$$

$$= f'(2)\cdot 2\cdot g(2) + f(2)\cdot g'(2)\cdot 3$$

$$= 4\cdot 2\cdot(-3) + 5\cdot(-5)\cdot 3$$

$$= -99$$

Suppose $g(x) = f^{-1}(x)$ and $f(x) = 2^x + 3x$. State the exact value for $g'(10)$. Justify your answer. (7 points)

Since $f(2) = 10$, then $f^{-1}(10) = g(10) = 2$.

$f'(x) = 2^x \ln x + 3$, so $f'(2) = 2^2 \ln x + 3$.

$$g'(10) = \frac{1}{f'(g(10))}$$

$$= \frac{1}{f'(2)}$$

$$= \frac{1}{4\ln 2 + 3}$$

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Suppose the curve, C, defined by $x^2 + xy - y^2 = 5$. Answer the following.

a. Verify that $\dfrac{dy}{dx} = \dfrac{2x+y}{2y-x}$. (7 points)

$$\frac{d}{dx}\left(x^2 + xy - y^2\right) = \frac{d}{dx}(5)$$

$$2x + 1\cdot y + x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$$

$$2x + y = (2y - x)\frac{dy}{dx}$$

$$\frac{2x+y}{2y-x} = \frac{dy}{dx}$$

b. Find an equation for each horizontal tangent line to C. (7 points)

$$\frac{dy}{dx} = 0 \quad \Rightarrow \quad 2x + y = 0 \quad \Rightarrow \quad x = -\frac{y}{2}$$

$$\left(-\frac{y}{2}\right)^2 + \left(-\frac{y}{2}\right)y - y^2 = 5 \quad \Rightarrow \quad y^2 = -4 \quad \Rightarrow \quad \textit{no solution}$$

c. Find an equation for each vertical tangent line to C. (7 points)

$$\frac{dy}{dx} \text{ is undefined} \quad \Rightarrow \quad 2y - x = 0 \quad \Rightarrow \quad y = \frac{x}{2}$$

$$x^2 + x\left(\frac{x}{2}\right)^2 - \left(\frac{y}{2}\right)^2 = 5 \quad \Rightarrow \quad x^2 = -4 \quad \Rightarrow \quad x = \pm 2$$

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Differentiate the following functions. Show the steps to your work and circle your final answer. (7 points each)

$w = \ln(\sin x)$

$w' = \frac{1}{\sin x} \cdot \cos x$

$= \cot x$

$z = \sec\theta \cdot \tan\theta$

$z' = \sec\theta \cdot \tan\theta \cdot \tan\theta + \sec\theta \cdot \sec^2\theta$

$= \sec\theta \cdot \tan^2\theta + \sec^3\theta$

$g(x) = \sin^{-1}\left(x^2\right)$

$g'(x) = \frac{1}{\sqrt{1-\left(x^2\right)^2}} \cdot 2x$

$= \frac{2x}{\sqrt{1-x^4}}$

$y = \frac{9}{\sqrt[3]{x^2-4x}} = 9\left(x^2-4x\right)^{-1/3}$

$y' = 9 \cdot \frac{-1}{3}\left(x^2-4x\right)^{-4/3} \cdot \left(2x-4\right)$

$= \frac{12-6x}{\left(x^2-4x\right)^{4/3}}$

$h(t) = \cos^3\left(3t^2\right)$

$h'(t) = 3\cos^2\left(3t^2\right) \cdot -\sin\left(3t^2\right) \cdot 6t$

$= -18t\cos^2\left(3t^2\right)\sin\left(3t^2\right)$

$y = \tan^{-1} x + \log\sqrt{x}$

$y' = \frac{1}{1+x^2} + \frac{1}{\sqrt{x} \cdot \ln 10} \cdot \frac{1}{2\sqrt{x}}$

$= \frac{1}{1+x^2} + \frac{1}{2x\ln 10}$

$y = x^{2x}$

$\ln y = 2x \ln x$

$\frac{1}{y} \cdot y' = 2 \cdot \ln x + 2x \cdot \frac{1}{x}$

$y' = y\left(2\ln x + 2\right)$

$y' = 2x^{2x}\left(1+\ln x\right)$

$f(x) = \frac{(x-4)^2}{(x+7)^5}$

$f'(x) = \frac{2(x-4)^1 \cdot 1 \cdot (x+7)^5 - (x-4)^2 \cdot 5(x+7)^4 \cdot 1}{\left[(x+7)^5\right]^2}$

$= \frac{(x-4)(x+7)^4\left[2(x+7) - 5 \cdot (x-4)\right]}{(x+7)^{10}}$

$= \frac{(x-4)(34-3x)}{(x+7)^6}$