AP CALCULUS (BC)
Evaluating Limits Analytically
Worksheet – KEY

Name ____________________
Date ____________________
Period ____________________

Evaluate the following limits.

1. $\lim_{w \to 0^-} \left(\frac{2+w}{e^w - 1} \right) = -\infty$

If you attempt a direct substitution, you get 2/0, which means the limit is unbounded. Since $x \to 0^-$, $e^w = 1 \to 0^-$. This means the denominator is negative while the numerator is positive. We can then conclude that the limit is unbounded in the negative direction.

2. $\lim_{y \to 0} \left(\frac{y}{\sqrt{y+3} - \sqrt{3}} \right) = \lim_{y \to 0} \left(\frac{y}{\sqrt{y+3} - \sqrt{3}} \cdot \frac{\sqrt{y+3} + \sqrt{3}}{\sqrt{y+3} + \sqrt{3}} \right) = \lim_{y \to 0} \left(\sqrt{y+3} + \sqrt{3} \right) = 2\sqrt{3}$

3. $\lim_{z \to -2^+} \sqrt{\frac{z^2 - 4}{2z^2 + z - 6}} = \sqrt{\lim_{z \to -2^+} \frac{(z-2)(z+2)}{(2z-3)(z+2)}} = \sqrt{\lim_{z \to -2^+} \frac{z-2}{2z-3}} = \sqrt{\frac{-4}{-7}} = \frac{2\sqrt{7}}{7}$

4. $\lim_{x \to \infty} \frac{\ln(x-1)}{\sqrt{x+2}} = 0$

As $x \to \infty$, both the numerator and denominator will increase without bound. The radical function will increase at a greater rate, however, so the denominator will get larger "faster" than the numerator. The result is a value that approaches zero.

5. $\lim_{t \to 2} \left(\frac{4t+3}{3t+27} \right) = \frac{11}{33} = \frac{1}{3}$

6. $\lim_{\alpha \to 0} \left(\frac{\sin 2\alpha}{\tan 4\alpha} \right) = \lim_{\alpha \to 0} \left(\frac{2}{4} \cdot \frac{\sin 2\alpha}{2\alpha} \cdot \frac{4\alpha}{\sin 4\alpha} \cdot \frac{\cos 4\alpha}{1} \right) = \frac{1}{2} \cdot 1 \cdot 1 \cdot 1 = \frac{1}{2}$

7. $$\lim_{c\to 0}\left(\frac{\frac{4}{c+6}-\frac{2+c}{3}}{\frac{c}{c+1}}\right)=\lim_{c\to 0}\left(\frac{\frac{12-(2+c)(c+6)}{3(c+6)}}{\frac{c}{c+1}}\right)=\lim_{c\to 0}\left(\frac{-c^2-8c}{3(c+6)}\cdot\frac{c+1}{c}\right)=\lim_{c\to 0}\frac{(-c-8)(c+1)}{3(c+6)}=\frac{-4}{9}$$

8. $$\lim_{q\to -\infty}\frac{\sqrt[3]{q-q^3}}{\sqrt{q^2+6q}}=\lim_{q\to -\infty}\frac{\sqrt[3]{-q^3}}{\sqrt{q^2}}=\lim_{q\to -\infty}\left(\frac{-q}{-q}\right)=1$$