Euler's identity states $e^{\pi i} + 1 = 0$.

This equation has historically been recognized for its mathematical beauty. It contains the three fundamental operations of addition, multiplication, and exponentiation. The identity consists of arguably the five most important numbers in mathematics – 0, 1, *e*, *π*, and *i*. In 2004, a poll given to mathematicians and scientists by *Physics World* concluded that Euler's identity was the "most important formula" ever discovered!

Below is an elegant proof of the identity using power series:

$$
\begin{aligned}
e^{\pi i} + 1 &= \left(\sum_{n=0}^{\infty} \frac{(\pi i)^n}{n!} \right) + 1 \\
&= \left(1 + \pi i + \frac{(\pi i)^2}{2!} + \frac{(\pi i)^3}{3!} + \frac{(\pi i)^4}{4!} + \frac{(\pi i)^5}{5!} + \frac{(\pi i)^6}{6!} + \frac{(\pi i)^7}{7!} + \cdots \right) + 1 \\
&= \left(1 + \pi i - \frac{\pi^2}{2!} - \frac{\pi^3}{3!} i + \frac{\pi^4}{4!} + \frac{\pi^5}{5!} i - \frac{\pi^6}{6!} - \frac{\pi^7}{7!} i + \cdots \right) + 1 \\
&= \left(1 - \frac{\pi^2}{2!} + \frac{\pi^4}{4!} - \frac{\pi^6}{6!} + \cdots \right) + \left(\pi i - \frac{\pi^3}{3!} i + \frac{\pi^5}{5!} i - \frac{\pi^7}{7!} i + \cdots \right) + 1 \\
&= \left(1 - \frac{\pi^2}{2!} + \frac{\pi^4}{4!} - \frac{\pi^6}{6!} + \cdots \right) + \left(\pi - \frac{\pi^3}{3!} + \frac{\pi^5}{5!} - \frac{\pi^7}{7!} + \cdots \right) i + 1 \\
&= \left(\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{(2n)!} \right) + \left(\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{(2n+1)!} \right) i + 1 \\
&= (\cos \pi) + (\sin \pi) i + 1 \\
&= -1 + 0i + 1 \\
&= 0
\end{aligned}
$$