1. What is the meaning of the expression $\lim_{\Delta x \to 0} \frac{\cos(x+\Delta x) - \cos x}{\Delta x}$?

The expression would find the function derivative of $f(x) = \cos x$.

2. If $g(x) = \sqrt{x}$, evaluate $g'(3)$.

$$\begin{aligned} g'(3) &= \lim_{\Delta x \to 0} \frac{\sqrt{3+\Delta x} - \sqrt{3}}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\sqrt{3+\Delta x} - \sqrt{3}}{\Delta x} \cdot \frac{\sqrt{3+\Delta x} + \sqrt{3}}{\sqrt{3+\Delta x} + \sqrt{3}} \\ &= \lim_{\Delta x \to 0} \frac{(3+\Delta x) - 3}{\Delta x\left(\sqrt{3+\Delta x} + \sqrt{3}\right)} \\ &= \lim_{\Delta x \to 0} \frac{1}{\sqrt{3+\Delta x} + \sqrt{3}} \\ &= \frac{1}{2\sqrt{3}} \end{aligned}$$

or

$$\begin{aligned} g'(3) &= \lim_{x \to 3} \frac{\sqrt{x} - \sqrt{3}}{x-3} \\ &= \lim_{x \to 3} \frac{\sqrt{x} - \sqrt{3}}{x-3} \cdot \frac{\sqrt{x} + \sqrt{3}}{\sqrt{x} + \sqrt{3}} \\ &= \lim_{x \to 3} \frac{x-3}{(x-3)\left(\sqrt{x} + \sqrt{3}\right)} \\ &= \lim_{x \to 3} \frac{1}{\sqrt{x} + \sqrt{3}} \\ &= \frac{1}{2\sqrt{3}} \end{aligned}$$

3. What is the line tangent to $f(x) = 3x^2 + 8$ at $x = -2$?

Using the alternative definition:

$$\begin{aligned} m &= f'(-2) \\ &= \lim_{x \to -2} \frac{\left(3x^2+8\right) - \left(3(-2)^2+8\right)}{x-(-2)} \\ &= \lim_{x \to -2} \frac{3x^2 - 12}{x+2} \\ &= \lim_{x \to -2} \frac{3(x-2)(x+2)}{x+2} \\ &= \lim_{x \to -2} \left(3(x-2)\right) \\ &= -12 \end{aligned}$$

Then the tangent line is $y_t = f(-2) + f'(-2)\left(x-(-2)\right) \quad \Rightarrow \quad y_t = 20 - 12(x+2)$.

4. Find the derivative of $y = x^3 - 2x$.

$$
\begin{aligned}
y' &= \lim_{\Delta x \to 0} \frac{\left[(x+\Delta x)^3 - 2(x+\Delta x)\right] - \left[x^3 - 2x\right]}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\left[x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 2x - 2\Delta x\right] - \left[x^3 - 2x\right]}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 2\Delta x}{\Delta x} \\
&= \lim_{\Delta x \to 0} \left(3x^2 + 3x\Delta x + (\Delta x)^2 - 2\right) \\
&= 3x^2 - 2
\end{aligned}
$$

5. What is the meaning of the expression $\lim_{t \to 3} \frac{4^t - 64}{t - 3}$?

The expression would find the numerical derivative of $f(t) = 4^t$ at $t = 3$.

6. Find the slope of the curve $y = \tan x$ at the origin.

Using the original definition:

$$
\begin{aligned}
m_{x=0} &= y'(0) \\
&= \lim_{\Delta x \to 0} \frac{\tan(0+\Delta x) - \tan 0}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\tan(\Delta x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \left( \frac{\sin(\Delta x)}{\Delta x} \cdot \frac{1}{\cos(\Delta x)} \right) \\
&= 1 \cdot 1 \\
&= 1
\end{aligned}
$$

7. If $h(y) = \frac{2}{y}$, compute $h'(y)$.

$$h'(y) = \lim_{\Delta y \to 0} \frac{\frac{2}{y + \Delta y} - \frac{2}{y}}{\Delta y}$$

$$= \lim_{\Delta y \to 0} \frac{2y - 2\left(y + \Delta y\right)}{\Delta y \cdot y \cdot \left(y + \Delta y\right)}$$

$$= \lim_{\Delta y \to 0} \frac{-1}{y \cdot \left(y + \Delta y\right)}$$

$$= \frac{-1}{y^2}$$